

3-4 Explore Compound Interest

Exercises

Round to the nearest cent where necessary.

- How much interest would \$2,000 earn in one year at the rate of 4.2%?
\$84
- How much interest would \$2,000 earn, compounded annually, in two years at the rate of 4.2%?
\$171.53
- How much interest would \$2,000 earn, with simple interest, in two years at the rate of 4.2%?
\$168.00
- Compare your answers to Exercises 2 and 3. Explain why they differ.
The compound interest is greater because the interest from the first year is added to the principal before computing the interest for year two.
- How much would d dollars earn in one year at the rate of p percent compounded annually?
$$\frac{dp}{100}$$
- Margaret deposits \$1,000 in a savings account that pays 5.4% interest compounded semi-annually. What is her balance after one year?
\$1,054.73
- How much interest does \$5,300 earn at a rate of 2.8% interest compounded quarterly, in three months?
\$37.10
- Mr. Guny deposits \$4,900 in a savings account that pays $3\frac{1}{2}\%$ interest compounded quarterly.
 - Find the first quarter's interest.
\$42.88
 - Find the first quarter's balance.
\$4,942.88
 - Find the second quarter's interest.
\$43.25
 - Find the second quarter's balance.
\$4,986.13
 - Find the third quarter's interest.
\$43.63
 - Find the third quarter's balance.
\$5,029.76
 - Find the fourth quarter's interest.
\$44.01
 - Find the fourth quarter's balance.
\$5,073.77
 - How much interest does the account earn in the first year?
\$173.77
- Jonathan deposits \$6,000 in a savings account that pays 3.2% interest compounded quarterly. What is his balance after one year?
\$6,194.32
- How much interest would \$1,000,000 earn at 5% compounded daily, in one day?
\$136.99

11. How much interest would y dollars earn in one day at a rate of 3.75% compounded daily?

$$\frac{0.0375y}{365}$$

12. Mrs. Huber opened a savings account on June 26 with a \$1,300 deposit. The account pays 3.6% interest compounded daily. On June 27, she deposited \$450 and on June 28 she withdrew \$110. Complete the table based on Mrs. Huber's banking activity.

	June 26	June 27	June 28
Opening balance	a. \$0	f. \$1,300.13	k. \$1,750.30
Deposit	b. \$1,300.00	g. \$450.00	---
Withdrawal	---	---	l. \$110.00
Principal used to Compute Interest	c. \$1,300.00	h. \$1,750.13	m. \$1,640.30
Interest	d. \$0.13	i. \$0.17	n. \$0.16
Ending Balance	e. \$1,300.13	j. \$1,750.30	p. \$1,640.46

13. Mr. Nolan has a bank account that compounds interest daily at a rate of 3.7%. On the morning of December 7, the principal is \$2,644.08. That day he withdraws \$550 to pay for a snow blower. Later that day he receives a \$934 paycheck from his employer, and he deposits that in the bank. On December 8, he withdraws \$300 to go holiday shopping. What is his balance at the end of the day on December 8?
\$2,728.67

14. Mrs. Platt has an account that pays p percent interest compounded daily. On April 27, she had an opening balance of b dollars. Also on April 27, she made a w dollars withdrawal and a d dollars deposit. Express her interest for April 27 algebraically.

$$\frac{p}{100}(b - w + d)$$

15. This morning, Mrs. Rullan had a balance of b dollars in an account that pays 3.05% interest compounded weekly. This afternoon she makes a withdrawal in the amount of w dollars. Express her interest for the day algebraically.

$$\frac{0.0305(b - w)}{365}$$

16. Kristin deposited \$9,000 in an account that has an annual interest rate of 4.1% compounded monthly. How much interest will she earn at the end of one month?
\$30.75

17. How much would \$25,000 earn in one hour at the rate of 5%, compounded hourly?
\$0.14

18. The Jules Server Scholarship Fund gives a graduation award of \$250 to a graduating senior at North End High School. Currently the fund has a balance of \$8,300 in an account that pays 5.2% interest compounded annually. Will the amount earned in annual interest be enough to pay for the award?
yes

19. Kelly has d dollars in an account that pays 3.4% interest compounded weekly. Express her balance after one week algebraically.

$$d + \frac{0.034d}{52}$$

3-5 Compound Interest Formula

Exercises

Round to the nearest cent wherever necessary.

- Mr. Mady opens a savings account with principal P dollars that pays 4.11% interest compounded quarterly. Express his ending balance after one year algebraically.
$$P\left(1 + \frac{0.0411}{4}\right)^4$$
- Jeff deposits \$2,300 at 3.13% interest compounded weekly. What will be his ending balance after one year?
\$2,373.11
- Nancy has \$4,111 in an account that pays 3.07% interest compounded monthly. What is her ending balance after two years?
\$4,370.98
- Mr. Weinstein has a savings account with a balance of \$19,211.34. It pays 4% interest compounded daily. What is his ending balance after three years, if no other deposits or withdrawals are made? How much interest does he earn over the three years?
\$21,660.58; \$2,449.24
- If you invested \$10,000 at 3.8% compounded hourly for five years, what would be your ending balance?
\$12,092.49
- Danielle has a CD at Crossland Bank. She invests \$22,350 for four years at 4.55% interest, compounded monthly. What is her ending balance? How much interest did she make?
\$26,802.15; \$4,452.15
- Ms. Santoro is opening a one-year CD for \$16,000. The interest is compounded daily. She is told by the bank representative that the annual percentage rate (APR) is 4.8%. What is the annual percentage yield (APY) for this account?
4.92%
- Knob Hill Savings Bank offers a one-year CD at 3.88% interest compounded daily. What is the APY for this account? Round to the nearest hundredth of a percent.
3.96%
- Kings Park Bank is advertising a special 5.08% APR for CDs. Kevin takes out a one-year CD for \$24,000. The interest is compounded daily. Find the APY for Kevin's account.
5.21%
- Imagine that you invest \$100,000 in an account that pays 5.9% annual interest compounded monthly. What will your balance be at the end of 18 years?
\$288,463.33
- Yurik invests \$88,000 in a CD that is locked into a 4.75% interest rate compounded monthly, for seven years. How much will Yurik have in the account when the CD matures?
\$122,631.08

12. Stephanie has created a study tool to help her study compound interest. She writes the compound interest formula with letters different than the traditional representations.

$$X = M\left(1 + \frac{Q}{K}\right)^{KB}$$

- a. If Q is increased, does the new balance increase or decrease? Explain your answer.
increase; interest rate increases
- b. If K is decreased, does the new balance increase or decrease? Explain.
decrease; less frequent compounding
- c. If B is increased, does the new balance increase or decrease? Explain.
increase; compounding for more years
- d. Is it possible that $M > X$? Explain.
No; the new balance is always greater (or the same) as the principal after interest is added.
- e. Using Stephanie's variable representation, express the amount of interest earned on the account.

$$X = M\left(1 + \frac{Q}{K}\right)^{KB} - M$$

13. Compare the simple interest for one year on a principal of 1 million dollars at an interest rate of 6.3% to compounding every second for the same principal and interest rate.

- a. How many seconds are in an hour?
3,600
- b. How many seconds are in a day?
86,400
- c. How many seconds are in a year?
31,536,000
- d. How much interest does \$1,000,000 earn in one year at 6.3% interest, compounded every second?
\$65,026.27
- e. How much does the same \$1,000,000 earn at 6.3% in one year, under simple interest?
\$63,000
- f. How much more interest did the compounded account earn when compared to the simple-interest account?
\$2,026.27

14. Britney invested \$4,000 in a CD at TTYL Bank that pays 3.4% interest compounded monthly.

- a. How much will Britney have in her account at the end of one year?
\$4,138.14
- b. What is the APY for this account? Round to the nearest hundredth of a percent.
3.45%

15. How much more would \$5,000 earn in ten years, compounded daily at 6%, when compared to the interest on \$5,000 over ten years, at 6% compounded semiannually?

\$79.58

3-6 Continuous Compounding

Exercises

Round to the nearest cent wherever necessary.

- Given the function $f(x) = \frac{1,234,999}{x}$, as the values of x increase towards infinity, what happens to the values of $f(x)$?
The values approach 0.
- As the values of x increase towards infinity, what happens to the values of $g(x) = 3x - 19$?
The values increase without bound.
- Given the function $h(x) = \frac{8x-3}{4x+5}$, as the values of x increase towards infinity, use a table to find out what happens to the values of $h(x)$.
The values approach 2.
- If $f(x) = \frac{10}{x^2}$, use a table and your calculator to find $\lim_{x \rightarrow \infty} f(x)$.
0
- Given the function $f(x) = 2^x$, find $\lim_{x \rightarrow \infty} f(x)$.
no limit
- Given the function $f(x) = \left(\frac{1}{2}\right)^x$, use a table to compute $\lim_{x \rightarrow \infty} f(x)$.
0
- If you deposit \$1,000 at 100% simple interest, what will your ending balance be after one year?
\$2,000

Compare simple interest with daily compounding and continuous compounding.

- If you deposit \$10,000 at 3.85% simple interest, what would your ending balance be after three years?
\$11,155
- If you deposit \$10,000 at 3.85% interest, compounded daily, what would your ending balance be after three years?
\$11,224.28
- If you deposit \$10,000 at 3.85% interest, compounded continuously, what would your ending balance be after three years?
\$11,224.35
- How much more did the account that was compounded continuously earn compared to the account compounded daily?
\$0.07
- How much more did the account that was compounded daily earn compared to the simple-interest account?
\$69.28

13. Eric deposits \$4,700 at 5.03% interest, compounded continuously for five years.

a. What is his ending balance?

\$6,043.98

b. How much interest did the account earn?

\$1,343.98

14. Write the verbal sentence that is the translation of $\lim_{x \rightarrow \infty} f(x) = 3.66$.

The limit of $f(x)$, as x approaches infinity, is 3.66.

15. Write the verbal sentence given below symbolically using limit notation.

The limit of $g(x)$, as x approaches zero, is fifteen.

$\lim_{x \rightarrow 0} g(x) = 15$

16. Given the function $f(x) = \frac{2x-17}{x}$, use a table to find $\lim_{x \rightarrow \infty} f(x)$.

2

17. Find the interest for each compounding period on \$50,000 for $2\frac{1}{2}$ years at a rate of 4.3%.

a. annually

\$5,549.58

b. semiannually

\$5,611.15

c. quarterly

\$5,642.61

d. monthly

\$5,663.85

e. daily

\$5,674.19

f. hourly

\$5,674.53

g. continuously

\$5,674.54

18. A private university has an endowment fund that currently has 49 million dollars in it. If it is invested in a one-year CD that pays 5.12% interest compounded continuously, how much interest will it earn?

\$2,574,135.57

19. Use a table of increasing values of x to find each of the following limits.

a. $\lim_{x \rightarrow \infty} f(x)$ if $f(x) = \frac{5x-2}{x+3}$

5

b. $\lim_{x \rightarrow \infty} g(x)$ if $g(x) = \frac{12x+5}{4x+3}$

3

c. $\lim_{x \rightarrow \infty} f(x)$ if $f(x) = \frac{5x^3-100}{x^2}$

∞

d. $\lim_{x \rightarrow \infty} f(x)$ if $f(x) = \frac{7x^2-1}{x^3+2}$

0

20. Find the interest earned on a \$14,000 balance for nine months at $3\frac{3}{4}$ % interest compounded continuously.

\$399.34

21. Assume you had P dollars to invest in an account that paid 5% interest compounded continuously. How long would it take your money to double? (Hint: Try substituting different numbers of years into the continuous compounding formula). Round to the nearest year.

14 years