

Independent Living

I long, as does every human being, to be at home wherever I find myself.

Maya Angelou, American Poet

8-1 Find a Place to Live

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8-3 Mortgage Application Process

8-4 Purchase a Home

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**What do you think
Maya Angelou meant in her quote?**

You may have heard the expressions “Be it ever so humble, there’s no place like home” and “Home is where the heart is.” A home can be many things. It can be a tent, an igloo, a hotel room, a tree house, an apartment, a condominium, a co-op, a house, a penthouse, or a mansion. A home is a place where a person establishes him or herself. The end of your schooling will mark a time in your life when you begin to become independent. When this happens, you may find that you will be spending a large part of your income on housing and household expenses. For most people, a home represents the largest financial undertaking of their lives. There are many factors that influence how and where you will live. With so much money involved, you will want to make wise, well-thought-out decisions. In this chapter, you will learn how to embark on the road to independent living.

Really?

Moving yourself to a new location is a big undertaking. But what about moving an entire building to a new location! This happens more often than you think.

Sometimes, prospective buyers fall in love with a home but not the location. Or, a developer is willing to sell an old structure for a cheap price so that he can build new structures on that location. Homes are not the only structures that take to the road. Here are the five heaviest buildings whose owners moved the buildings to different locations.

- The Shubert Theater, Minneapolis, Minnesota—2,908 tons
- The Hotel Montgomery, San Jose, California—4,816 tons
- Cape Hatteras Lighthouse, Outer Banks, North Carolina—4,830 tons
- Newark International Airport Building 51, Newark, New Jersey—7,400 tons
- Fu Gang Building, Guangxi Province, China—15,140 metric tons

Moving a building to a new location takes a great deal of mathematics. Math plays a central role in the planning, engineering, approvals, finances, and more.



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Really!

What is more agreeable than one's home?

Marcus Tullius Cicero, Ancient Roman Writer, Scholar, and Statesman

8-1

Find a Place to Live

Objectives

- Calculate the affordability of a monthly rent.
- Determine the relationship between square footage and monthly rent.
- Determine lease signing costs.
- Calculate moving expenses.

Key Terms

- apartment
- tenant
- landlord
- furnished
- unfurnished
- lease
- expire
- evict
- single-family home
- square footage
- application deposit
- security deposit

WHERE WILL YOU LIVE?

Have you ever imagined what it would be like to have a place of your own? For many teenagers, the usual progression of living arrangements is from family home to dorm room to apartment to homeownership. Finding a place to live isn't easy. There are many decisions to make.

Your first experience in independent living will probably be in a rented **apartment**. When you rent an apartment, you are the **tenant**, and the owner of the apartment is the **landlord**. As you look for an apartment to rent, you will see that they come **furnished** or **unfurnished**. The cost of renting a furnished apartment includes the use of the landlord's furniture in that apartment. You must provide your own furniture when you rent an unfurnished apartment. Before you move into any apartment, you must sign a **lease**. A lease is a written agreement between the landlord and the tenant that details the amount of rent and the length of time that you will rent the apartment. The lease states the rules and regulations that must be followed by the tenant and the landlord. After a lease **expires** or ends, the tenant may sign a new lease for a new period of time and this lease may have an increase in the rent. If, for any reason, a tenant stops paying rent, they have defaulted on the lease, and may be **evicted** from the apartment.

While renting is a suitable option for many, others find owning a home is their goal. There are many options for ownership. You can purchase a **single-family home**, a multiple-family home, a condominium, or a cooperative.

Rather than renting an apartment, there is a possibility that you can purchase and own that apartment if it is part of a cooperative or condominium. A condominium is a form of home ownership where each unit is individually owned. Each individual unit is called a condominium or condo. Condominium owners own everything from the walls inward and are responsible for the maintenance of the inside of their own units. The owners are charged a maintenance fee that is used to maintain common areas such as a lobby, lawn, roof, sidewalks, and roads.

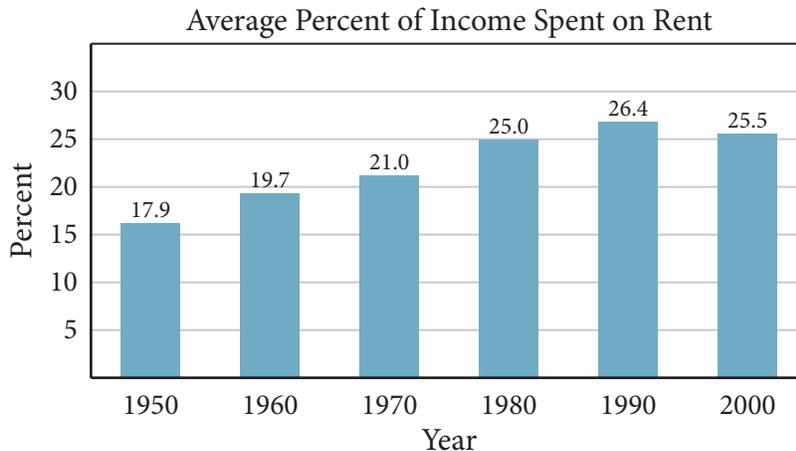
A co-op apartment or residence is another form of home ownership. A cooperative is a corporation. Cooperative owners own shares in the corporation and the right to live in a unit. They are also responsible for the maintenance of the inside of their units.

Skills and Strategies

Here you will learn how to make sound decisions when considering rentals or purchases based on data available.

Affording the Rent

Every 10 years, the U.S. Census Bureau collects data. The following chart illustrates the average amount that people paid for rent as a percentage of their incomes.



Notice how the percentage appears to be hovering around 25% since 1980. Experts agree that as a rule, a prospective renter should budget 25% to 30% of their gross income for rent.

EXAMPLE 1

Alex makes \$61,992 per year and pays about 25% of his gross monthly income in federal and state taxes. He wants to find an apartment to rent. Estimate how much he can afford to pay for rent each month. Then determine how much money he will have after taxes and rent are paid.

SOLUTION The recommended rule is to budget 25% to 30% of the gross income for rent. A good estimate to use is 28%. Rent is paid on a monthly basis, so find Alex's gross monthly income.

Divide annual income by 12. $61,992 \div 12 = 5,166$

Alex's gross monthly income is \$5,166. Find 28% of his monthly income to estimate an affordable amount.

Multiply by 0.28. $5,166 \times 0.28 = 1,446.48$

Alex can afford an apartment with a monthly rent of about \$1,446.

Alex pays 25% of his gross monthly income in federal and state taxes.

Multiply by 0.25. $5,166 \times 0.25 = 1,291.50$

Alex pays about \$1,291 in taxes each month. To find the amount remaining each month, subtract the amounts for rent and taxes from Alex's monthly income.

$$5,166 - 1,446 - 1,292 = 2,430$$

Alex will have approximately \$2,430 remaining.

- You could also find the remaining amount by determining the percent of Alex's income that is not spent on taxes and rent. Subtracting these percents from 100% yields 47% since $100 - 25 - 28 = 47$. You can verify that 47% of \$5,166 equals \$2,428.02, which is close to the estimate found previously.

■ CHECK YOUR UNDERSTANDING

Bethany's monthly gross income is \$3,840. She pays 24% of her monthly gross earnings in federal and state taxes and 15% for her student loan. Bethany uses 15% of her monthly gross income to pay toward her credit card balance. She wants to rent an apartment that will cost \$1,800 per month. Will she be able to make the payments without changing the amounts she pays toward her student loans and credit card balances?

Shopping for a Rental

Once you have determined what you can afford to pay in rent, it is time to start looking at the classified ads for rental property. As with automobile classifieds, you should become familiar with the abbreviations that are used. Here are a few of the common ones.

ba or bth, bathroom	renov, renovated
br, bedroom	rm, room
DW or D/W or dshwr, dishwasher	stu, studio
DR, dining room	spac, spacious
Drmn, doorman	WIC, walk-in closet
EIK, eat-in-kitchen	W/D, washer and dryer
elev, elevator in building	w/d hkup, washer and dryer hookup
gar, garage	w/w, wall-to-wall carpeting
h/w, hardwood floors	yd, yard
htd, heat is included in rent	
incl ht/hw, includes heat and hot water	
mint, excellent condition	



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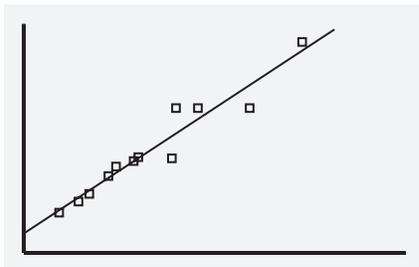
In addition to explaining what the apartment has to offer, classified ads often include the **square footage** of each apartment. This number is the amount of floor space available in the apartment.

EXAMPLE 2

Rufus and Maria have both been offered new jobs in a different city. A real estate broker sent them a listing of apartments in their desired location showing the square footage in each apartment. Use linear regression analysis to determine if there is a correlation between the square footage of rental property and the amount charged for the monthly rent. What is the linear regression equation? Interpret the correlation coefficient.

SOLUTION Use the statistics features on your graphing calculator to create a scatterplot. Graph the linear regression line.

Square Feet	Monthly Rent (\$)
664	995
735	1,045
787	1,095
872	1,205
903	1,245
993	1,325
976	1,295
1,133	1,295
1,150	1,595
1,244	1,595
1,474	1,595
1,697	1,995



```
LinReg
y=ax+b
a=.9061573426
b=403.0870898
r2=.9229477794
r=.960701712
```

The linear regression equation is $y = 0.91x + 403.09$ to the nearest hundredth.

The correlation coefficient of 0.96 indicates that square footage is a good predictor of the amount charged as rent for these apartments.

■ CHECK YOUR UNDERSTANDING

Based on Example 2, what is a good estimate for the amount of monthly rent charged for an 880-square foot apartment?

Lease Signing Costs

It isn't enough just to have the first month's rent available. There are a number of fees that are associated with the rental of any property. Usually, there is an **application deposit**. This amount, which is sometimes refundable, may vary between \$100 and \$400. It covers the cost of processing the application for the rental. Often a credit report is required. The fee for this report is usually under \$25.

A **security deposit** is money given to the landlord from the tenant as protection in the event that the tenant causes damage to the rented property. This deposit is refunded when the tenant moves out if there is no damage. The security deposit can range from 1 to 4 month's rent.

In addition to the first month's rent paid in advance, many landlords also require the last month's rent to be paid at the time of move in. This protects the landlord in the event that the tenant decides to break the lease and vacate the apartment earlier than agreed in the contract.

If you use a broker to find an apartment, there will be an additional fee for the broker's services; usually a percentage of a year's rent.

EXAMPLE 3

Rufus and Maria paid a \$200 application deposit for the 1,150-square foot apartment in Example 2. They are required to provide a credit report that costs \$25 and pay a security deposit equal to one month's rent. The landlord also requires the last month's rent at the time of signing the lease. The broker charged 10% of the yearly rent. How much should they expect to pay to be able to move into the apartment?

SOLUTION Rufus and Maria should plan on paying the following:

Application deposit	\$ 200
Credit report fee	\$ 25
Security deposit: 1 month's rent	\$1,595
Last month's rent	\$1,595
Broker's fee: $0.1(1,595 \times 12)$	\$1,914
First month's rent	\$1,595
Add these amounts to find the total.	\$6,924

Rufus and Maria should expect to pay \$6,924 before moving into their apartment.

■ CHECK YOUR UNDERSTANDING

Larry is renting an apartment that will cost r dollars per month. He must pay a \$100 application fee and a \$25 credit report fee. His security deposit is two month's rent, and he must also pay the last month's rent upon signing the lease. His broker charges 5% of the total year's rent as the fee for finding the apartment. Express in terms of r the total cost of signing the lease.

Moving Costs

Whether you are renting or purchasing a home, you need to budget for moving expenses. When planning a move, you should consider all of the options available to you.

You can elect to have someone do all of the packing, loading, transporting, unloading, and unpacking for you. Or, you can do all or part of it yourself, with or without help from professionals.

The cost of making a move depends upon a variety of factors: how much of the work you choose to do, the distance you are moving, the weight and size of your belongings, how accessible the items are to street level (Are you on the first floor? Are there many flights of stairs? Is there an elevator?), and the location of the pick up and drop off of your items. These are only a few of the factors that come into play when you are given a moving estimate. Many companies offer online services to help you.

EXAMPLE 4

Jay is moving from an apartment in Miami to one in Orlando. If Jay moves on a weekday, he will need more movers' time to pack, load, unload, and unpack because his friends will not be able to help him. If he moves on a weekend, he can get his friends to help, cutting down on the number of hours he will need to hire movers. MoveOut is a moving company that supplies movers, trucks, and moving equipment. They have given him the following moving estimates.

Weekday Move	Weekend Move
6 hours of loading/unloading	4 hours of loading/unloading
5 hours of packing/unpacking	2 hours of packing/unpacking
\$720 total cost	\$400 total cost

MoveOut charges a set hourly moving team rate for loading and unloading, and a different set hourly moving team rate for packing and unpacking. Determine the MoveOut hourly rates.

SOLUTION Solve this problem by setting up a system of two equations. First, identify the variables to use. Let x represent the hourly cost for loading/unloading. Let y represent the hourly cost for packing/unpacking.

Two equations can be written that model the moving costs.

$$\text{Weekday Move } 6x + 5y = 720 \qquad \text{Weekend Move } 4x + 2y = 400$$

To graph these linear equations, first solve for y .

$$6x + 5y = 720$$

$$5y = -6x + 720$$

$$\frac{5y}{5} = \frac{-6x + 720}{5}$$

$$y = -\frac{6}{5}x + 144$$

$$4x + 2y = 400$$

$$2y = -4x + 400$$

$$\frac{2y}{2} = \frac{-4x + 400}{2}$$

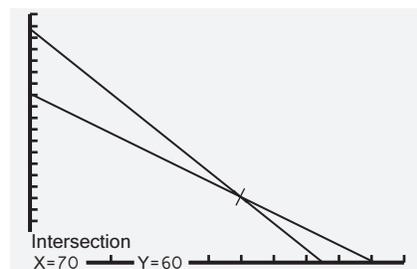
$$y = -2x + 200$$

WINDOW
Xmin=1
Xmax=130
Xscl=10
Ymin=0
Ymax=210
Yscl=10
Yres=1

Use values slightly greater than the x - and y -intercepts to determine an appropriate viewing window. In the weekday equation, the x -intercept is 120, and the y -intercept is 144. In the weekend equation, the x -intercept is 100, and the y -intercept is 200.

The setting for the viewing window and the graphs of the two equations are shown.

Using the intersection feature on your calculator, you can determine that the two lines intersect at the point (70, 60). This indicates that MoveOut charges \$70 per hour for loading/unloading and \$60 per hour for packing/unpacking.



■ CHECK YOUR UNDERSTANDING

Using the information above, suppose that Jay hired the movers for P hours to pack and unpack and for L hours to load and unload. Write an expression that represents his moving cost for these services.

Systems of equations can also be solved algebraically. The *elimination method* is the process of algebraically manipulating one or both equations so that the coefficients of one set of variable terms are opposite and will drop out when the equations are combined as shown in Example 5.

EXAMPLE 5

Samantha is moving from Madison, WI to La Crosse, WI. She will do all of the packing and unpacking by herself with her brother. The moving company quoted a price of \$1,250 for 8 hours of loading and unloading and driving 130 miles. The company quoted the same price if the truck drives an extra 30 miles to pick up Samantha's brother. Samantha figures that with her brother's help she only needs to hire the movers for 6 hours. How much does the company charge per hour for the loading/unloading? How much do they charge per mile for driving?

SOLUTION Let x represent the hourly cost for loading and unloading. Let y represent the per-mile cost for renting the truck.

	Load/Unload Hours	Mileage
Situation without brother	8	130
Situation with brother	6	160
Cost	\$1,250	\$1,250

Write the equations that model each situation.

Labor cost + Truck rental cost = Total cost

$$8x + 130y = 1,250$$

$$6x + 160y = 1,250$$

Use the elimination method to solve the system of equations. Multiply the first equation by -3 , and multiply the second equation by 4 so the coefficients of the x -terms are opposites.

$$-3(8x + 130y) = -3(1,250) \rightarrow -24x - 390y = -3,750$$

$$4(6x + 160y) = 4(1,250) \rightarrow \underline{24x + 640y = 5,000}$$

Add the equations.

$$250y = 1,250$$

Solve for y . Divide each side by 250.

$$y = 5$$

Use $y = 5$ to substitute into either of the original equations to solve for x .

First equation

$$8x + 130y = 1,250$$

Substitute 5 for y .

$$8x + 130(5) = 1,250$$

Simplify.

$$8x + 650 = 1,250$$

Subtract 650 from each side.

$$8x = 600$$

Divide by 8.

$$x = 75$$

Samantha will pay \$75 per hour for loading/unloading and \$5 per mile for the truck rental.

■ CHECK YOUR UNDERSTANDING

If you graph the two equations in Example 5, what is the point of intersection?

Applications

What is more agreeable than one's home?

Marcus Tullius Cicero, Ancient Roman Writer, Scholar, and Statesman

1. Explain how this quote can be interpreted in light of what you have learned.
2. Use the interval 25%–30% to find the monetary range that is recommended for the monthly housing budget in each situation. Round to the nearest dollar.
 - a. Mark makes \$86,000 per year.
 - b. Linda makes \$7,000 per month.
 - c. Meghan makes \$1,500 per week.
3. Jessica's financial advisor believes that she should spend no more than 28% of her gross monthly income for housing. She has determined that amount is \$1,400 per month. Based on this amount and her advisor's recommendation, what is Jessica's annual salary?
4. Abe makes \$18.50 per hour. He works 37 hours a week. He pays 23% of his gross earnings in federal and state taxes and saves 5% of his monthly gross income. He is considering renting an apartment that will cost \$1,500 per month.
 - a. Is this monthly rental fee within the recommended 25%–30% housing expense range?
 - b. Based upon his expenses, can he make the monthly payments?
5. Rachel is considering moving into a one-bedroom apartment in Glen Gardens. The apartment has a monthly rent of \$1,300. Here are the fees that she has been quoted. How much is she expected to pay up front in order to rent this apartment?
6. Milena has a gross biweekly income of \$2,200. She pays 18% in federal and state taxes, puts aside 10% of her income to pay off her school loan, and puts 5% of her income aside for savings. She is considering an apartment that rents for \$1,200 per month.
 - a. Is this monthly rental fee within the recommended 25%–30% housing expense range?
 - b. Based on her expenses, can she make the monthly payments?
7. A moving helper company gave Mike these two quotes. Use a system of equations to determine the hourly rates for loading/unloading and packing/unpacking.
8. Jaden received these two estimates from a moving company. Write and solve a system of equations to determine the hourly loading/unloading fee and the mileage charge for the truck rental.

Application fee: 2% of one month's rent
Credit application fee: \$10
Security deposit: 1 month's rent
Last month's rent
Broker's fee: 12% of one year's rent

3 hours of loading/unloading	5 hours of loading/unloading
2 hours of packing/unpacking	2 hours of packing/unpacking
Total cost: \$480	Total cost: \$680

Situation A: He hires 5 helpers to load and unload the truck and travels 80 miles on back roads for a total cost of \$780.
Situation B: He hires 6 helpers to load and unload the truck and takes a highway route which adds 20 miles to the trip but gets the truck to the destination faster for a total cost of \$960.

Square Feet	Monthly Rent	Square Feet	Monthly Rent
400	\$ 980	500	\$1,200
1,000	\$2,000	700	\$1,600
650	\$1,500	900	\$1,900
800	\$1,700	750	\$1,550
850	\$1,725	480	\$1,050

9. Ann obtained this list of apartments.
- Use linear regression analysis to determine if there is a correlation between the square footage and the monthly rent.
 - Determine the regression equation. Round the numbers in the equation to the nearest hundredth.
 - Use your regression equation to determine the price you might expect to pay for an 810-square foot apartment.

10. Use the information from Exercise 9.
- Determine the correlation coefficient and linear regression equation that expresses the square footage as a function of the monthly rent. Round the numbers in the equation to the nearest hundredth.
 - Use your regression equation to determine the square footage you might expect if renting a \$1,710 apartment.

Application Fee: 1.5% of one month's rent
 Credit Application Fee: \$10
 Security Deposit: 1 month's rent
 Last month's rent
 Broker's Fee: 9% of one year's rent

11. Dave wants to rent a two-bedroom apartment in City Fields. The apartment has a monthly rent of D dollars. Here are the fees that he has been quoted. Write an algebraic expression that represents the amount he is expected to pay before renting the apartment.

12. The square footage and monthly rental of 10 similar one-bedroom apartments yield the linear regression $y = 0.775x + 950.25$, where x represents the square footage of the apartment and y represents the monthly rental price. Grace can afford \$1,500 per month rent. Using the equation, what size apartment should she expect to be able to rent for that price?
13. The square footage and monthly rental of 10 similar two-bedroom apartments yield the linear regression formula $y = 1.165x + 615.23$ where x represents the square footage of the apartment and y represents the monthly rental price.
- Use the formula to determine the monthly rent for an apartment that has 1,500 square feet.
 - Based upon the recommendation that you should spend no more than 28% of your monthly gross income on housing, can Jacob afford this rental if he makes \$8,000 each month. Explain.

\$85 per hour for loading/unloading service
 \$70 per hour for packing/unpacking service
 \$5 per mile for truck rental

14. WeMoveU charges for moving according to the rate schedule shown. Nicky is moving a distance of 150 miles and needs 7 hours of loading/unloading and 5 hours of packing/unpacking. What will her moving cost be if the service also charges 8% tax on the total?

L dollars per hour for loading and loading service
 P dollars per hour for packing and unpacking service
 M dollars per mile for truck rental

15. Van4Hire charges for moving according to this rate schedule. Nicky is moving a distance of D miles and needs A hours of loading/unloading and B hours of packing/unpacking. Write an algebraic expression that represents her total moving cost.

A good home must be made, not bought.

Joyce Maynard, American Author

Read a Floor Plan

8-2

Key Terms

- floor plan
- area
- congruent
- apothem
- perimeter
- Monte Carlo method
- volume
- British Thermal Units (BTUs)

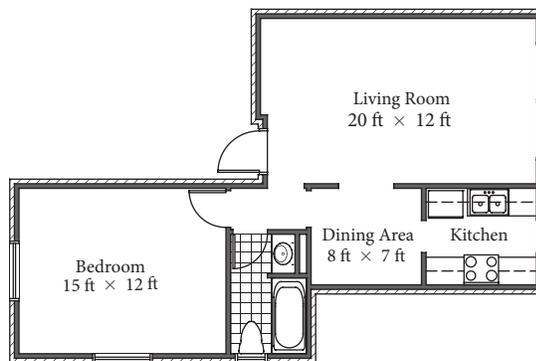
Objectives

- Compute the perimeter and the area of a polygon.
- Compute areas of irregular regions.
- Compute volumes of rectangular solids.

HOW MUCH SPACE DO I WANT? HOW MUCH SPACE DO I NEED?

When you begin to look for a place to live, price and location are not the only factors you should consider. One important consideration is the number of rooms, room sizes, and the layout of the living space. This information is shown on a floor plan. A **floor plan** is a drawing of the layout and dimensions of rooms. The floor plan shown is for a one-bedroom apartment. Here is some of the information that the floor plan shows you:

- This apartment has a kitchen, a dining room, a living room, a bedroom, and a bathroom.
- The bathroom has a sink, a toilet, a bathtub, and one window.
- The bedroom is 15 feet by 12 feet; it has two windows and one doorway.
- The living room is 20 feet by 12 feet; it has one large window and three entrances, one being the outside entrance.
- The dining area is 8 feet by 7 feet and has three entrances and no windows.
- The kitchen has cabinets, counter space, a sink, a stove, a refrigerator, one window, and one entrance.



Examining a floor plan allows you to make intelligent decisions before you move in. It helps you determine what furniture will fit in each room. Everybody likes to have ample space that suits their living style, family size, and hobbies. Larger spaces usually are more expensive, so deciding what you want and what you need is a balancing act with what you can afford.

You can create floor plans by hand or by using computer software specifically designed to draw floor plans. Floor plans are drawn to scale—they are drawn in the same proportion as the actual home. Even if you never need to draw your own floor plan, you need to know how to read them.

Skills and Strategies

Here you will learn to read floor plans and use information in the floor plans to make buying decisions.

EXAMPLE 1

Jerry is using the floor plans for his new home to help him purchase base molding for the place where the walls meet the floor. The plans are drawn using a scale of $\frac{1}{4}$ inch represents 1 foot.

He measures the walls on the floor plan with a ruler and finds that they total $23\frac{1}{2}$ inches. If molding costs \$2.10 per foot, how much will Jerry spend on molding?

SOLUTION Jerry needs to use a proportion to convert the scale measurements to the actual measurements in feet. Recall that a proportion is created when two ratios are set equal to each other.

When setting up a proportion, keep like units in the numerators and like units in the denominators.

$$\frac{0.25 \text{ inch}}{1 \text{ foot}} = \frac{\text{Number of inches on the floor plan}}{\text{Actual number of feet in the room}}$$

Substitute.
$$\frac{0.25 \text{ inch}}{1 \text{ foot}} = \frac{23.5 \text{ inches}}{x}$$

Cross multiply.
$$0.25x = 23.5(1)$$

Divide each side by 0.25.
$$x = 94$$

Jerry needs 94 feet of molding.

To find the total cost, multiply the number of feet of molding by the cost per foot.

$$94(2.10) = 197.40$$

The molding will cost Jerry \$197.40.



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■ CHECK YOUR UNDERSTANDING

The length of a room is 17 feet. When using $\frac{1}{4}$ inch = 1 foot scale, what is the length of the room on a floor plan?

EXAMPLE 2

Gabriela plans to carpet her living room, except for the quarter-circle shown in the corner. That area will be a wood floor where she will put her piano. The radius of the quarter circle is 8 feet. If carpeting costs \$9.55 per square foot, what is the cost of the carpeting she will use in her living room?

SOLUTION Find the **area** of the entire rectangle, and then subtract the area of the quarter circle to find the area of the carpeted section of the floor. In the formula for area, the length is l and the width is w .

Formula for area $A = lw$

Substitute and multiply. $A = 25(16) = 400$

The area of the room is 400 square feet.

The area of a circle with radius r is given by the formula

$$A = \pi r^2$$

Therefore, the area of the quarter-circle is given by the formula

$$A = \frac{1}{4} \pi r^2$$

Substitute 8 for r and 3.14 for π . $A = \frac{1}{4}(3.14)(8)^2$

Simplify. $A = 0.25(3.14)(64) = 50.24$

The area of the quarter-circle is approximately 50.24 square feet.

Subtract to find the area of the carpeted region. $400 - 50.24 = 349.76$

Gabriela needs approximately 350 square feet of carpeting.

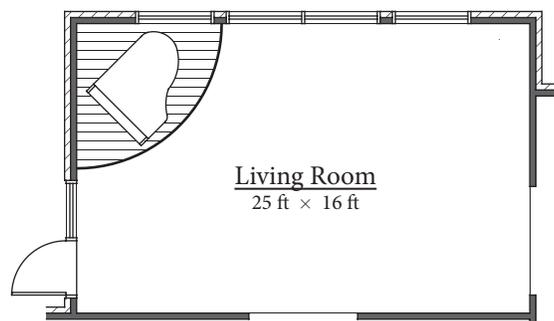
Multiply the number of square feet needed by \$9.55 to find the total cost of the carpeting that will be used.

$$350(9.55) = 3,342.50$$

The carpeting will cost approximately \$3,342.50.

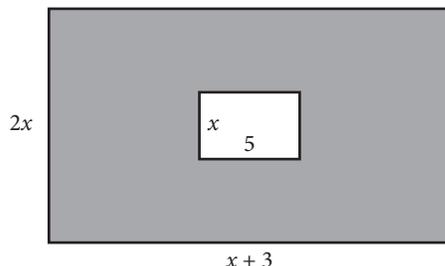
Due to the shape of the carpeted area, Gabriela may have to purchase a few extra square feet of carpet to make sure the installers have enough.

This is determined at the time of purchase.



■ CHECK YOUR UNDERSTANDING

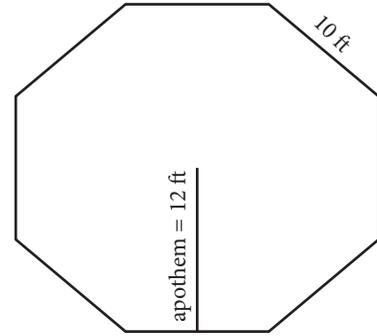
Express the area of the shaded region in the room shown algebraically.





EXAMPLE 3

Delgado's Landscape Design is building a large gazebo for a backyard. It is in the shape of a regular octagon as shown in the diagram. Each side of the gazebo is 10 feet. They need to purchase wood for the floor. It costs \$14 per square foot for a special type of wood. Find the cost of the gazebo's floor.



SOLUTION All eight sides of the regular octagon are **congruent**, so they have the same length. To find the area of a regular octagon, you need its apothem and its **perimeter**. The **apothem**, as shown in the diagram, is a line segment through the center of a regular polygon that is perpendicular to a side.

$$A = \frac{1}{2}ap \quad \text{where } A \text{ is the area of the regular polygon, } a \text{ is the apothem, and } p \text{ is the perimeter}$$

The perimeter is the product of the number of sides and the length of each side.

An octagon has 8 sides. $p = 8(10) = 80$

The apothem is 12 feet long.

Substitute and simplify. $A = \frac{1}{2}ap$

$$A = \frac{1}{2}(12)(80) = 480$$

The area is 480 square feet.

Multiply the area by the price per square foot to obtain the total cost.

Square feet \times cost per foot $480(14) = 6,720$

The flooring will cost \$6,720.

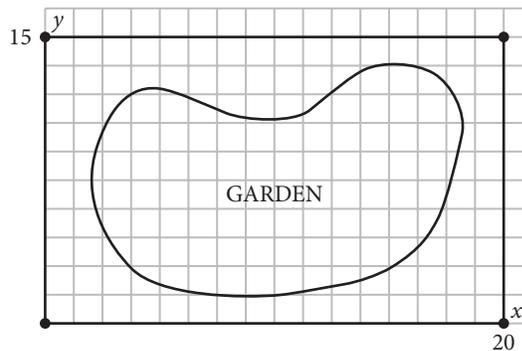
■ CHECK YOUR UNDERSTANDING

A regular pentagon (5 sides) has an area of 440 square units, and each side measures x units. Express the apothem of the pentagon algebraically in terms of x .

EXAMPLE 4

Don sculpts out a region for a flower garden, as shown. He takes a digital picture of the garden. The irregular region would fit inside a rectangle that is 15 yards by 20 yards. He superimposes a 15 by 20 grid over the photo on his computer. The area of the garden impacts the cost of mulch, plants, fertilizer, and so on. What is the area of the garden?

SOLUTION The garden is not a polygon. It is a free-form plane figure. Don can use probability to find the area. First, he frames the irregular region with a 15 by 20 rectangle, whose area is 300 square units. Then, he generates 1,000 random points (x, y) with a domain $0 \leq x \leq 20$ and a range $0 \leq y \leq 15$. Most graphing calculators have a random number feature that can generate x - and y -values for this problem.



Don plots the points and determines the number of points that landed inside the garden region. He finds that 631 points landed inside the garden region. He uses the following proportion.

$$\frac{\text{Number of points inside the region}}{\text{Total number of random points}} = \frac{\text{Area of irregular region}}{\text{Area of rectangle used to frame irregular region}}$$

Substitute. $\frac{631}{1,000} = \frac{x}{300}$

Cross multiply. $1,000x = 631(300)$

Multiply. $1,000x = 189,300$

Solve for x . $x = 189.3$

The area of the free-form garden is approximately 189 square feet.

The method used in this example is the **Monte Carlo method**. It is based on probability. The theory is that the ratio of points that land inside the region to the total points should equal the ratio of the area of the irregular region to the area of the rectangle. The more points you use, the more accurate your approximation.

■ CHECK YOUR UNDERSTANDING

An irregular plane figure is framed inside of a 20 by 20 square that represents a 20-foot by 20-foot square. To find its area, 2,000 random points are generated, and 910 of them land inside the irregular region. What is the area of the irregular region, to the nearest integer?

Volume is the amount of space inside a three-dimensional region, such as a room. Volume is measured in cubic units. Most rooms are in the shape of rectangular solids and have length, width, and height.

EXAMPLE 5

Find the volume of a room 14 feet by 16 feet with an 8-foot ceiling.

SOLUTION The formula for the volume of a rectangular solid with length l , width w , and height h , is

$$V = lwh$$

Substitute and simplify. $V = (16)(14)(8) = 1,792$

The volume of the room is 1,792 cubic feet.

■ CHECK YOUR UNDERSTANDING

A square room with side x units long has volume 1,900 cubic units. Express the height of the ceiling algebraically in terms of x .

EXAMPLE 6

Mike's bedroom measures 16 feet by 14 feet, and has a 9-foot ceiling. It is well-insulated and on the west side of his house. How large of an air conditioner should he purchase?

SOLUTION Air conditioners are sold according to their **BTU (British Thermal Units)** rating. The air conditioner with the right BTU rating will cool and dehumidify a room. If the rating is too low, the room may not cool sufficiently. If the rating is too high, it may shut off before removing the humidity. Many appliance salespeople use the formula known as "while divided by 60" to compute the correct BTU rating.

In this formula, w represents the width of the room, h represents the height, and l represents the length. The level of insulation is represented by i . If the room is well-insulated, $i = 10$, and if the room is poorly insulated, $i = 18$. The variable e represents the exposure—the direction the outside wall of the room faces. If it faces north, $e = 16$. If it faces east, $e = 17$. If it faces south, $e = 18$, and if it faces west, $e = 20$.

$$\text{BTU rating} \approx \frac{\text{while}}{60}$$

Substitute. $\text{BTU rating} \approx \frac{(14)(9)(10)(16)(20)}{60}$

Simplify. $\text{BTU rating} \approx 6,720$

Mike should purchase an air conditioner rated at 7,000 BTUs.

■ CHECK YOUR UNDERSTANDING

In Example 6, find the recommended BTUs if Mike's room was poorly insulated, and the other variables remained the same.

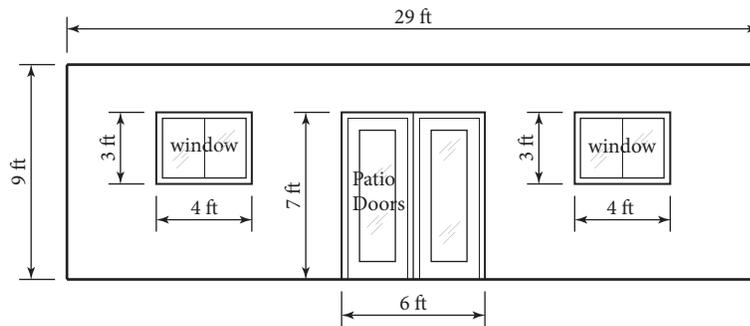
Applications

A good home must be made, not bought.

Joyce Maynard, American Author

1. Interpret the quote in the context of what you think is the difference between a house and a home.
2. A rectangular room measures 13 feet by 15 feet. It is going to be carpeted with carpeting that sells for \$68 per square yard.
 - a. What is the area of the room in square feet?
 - b. If the room is drawn to $\frac{1}{4}$ inch represents 1 foot scale, give the dimensions of the room in inches on the scale drawing.
 - c. A yard equals 3 feet. How many square feet are in a square yard?
 - d. How many square yards will it take to carpet the room? Round up to the next greater integer.
 - e. What is the total price for the carpeting?
 - f. In the early mid-2000s, there was a dramatic increase in the price of carpeting, since it was a petroleum-based product. As a result, selling by the yard made it look expensive. Some stores started selling carpet by the square foot. How much would a square yard of carpet selling for \$8.95 per square foot cost?
 - g. This room is well-insulated and is on the south side of the house. It has an 8-foot-high ceiling. How large an air conditioner would this room require? Round to the nearest thousand BTUs.
3. A rectangular room has length L and width W , where L and W are measured in feet.
 - a. Express the area in square feet algebraically.
 - b. If carpeting costs x dollars per square yard, express the cost of carpeting this room algebraically.
4. A rectangular room is 14 feet by 20 feet. The ceiling is 8 feet high.
 - a. Find the length and width of the smaller wall.
 - b. Find the area of the smaller wall.
 - c. Find the area of the larger wall.
 - d. Find the total area of the four walls in the room.
 - e. If a gallon of paint costs \$36.50 and it covers 350 square feet on average, what is the cost of painting the room with two coats of paint? Explain your answer.
 - f. This room is well-insulated and is on the north side of the house. How large an air conditioner would this room require? Round to the nearest thousand BTUs.
5. A gazebo in the shape of a regular octagon has equal sides of 9 feet and an apothem of 10.9 feet.
 - a. If one side of a gazebo is open, and the other sides have a railing, find the cost of the railing if it sells for \$7.90 per foot.
 - b. Find the area of the gazebo.
 - c. Find the cost of the gazebo's floor if the flooring costs \$3 per square foot. Round to the nearest hundred dollars.

6. A rectangular room measures x feet by y feet, where $x < y$. The ceiling is c feet high. The walls and the ceiling will be painted the same color.
- Express the area of the smaller wall and the larger wall algebraically.
 - Express the total area of the room algebraically.
 - Express the area of the ceiling algebraically.
 - If a gallon of paint covers 400 square feet, express the amount of paint you will need algebraically.
7. A gazebo in the shape of a regular hexagon has side length s and apothem a . If the cost per square unit of flooring is c , express the total cost T of the floor algebraically.
8. Giuliana is having one wall of her den covered in brick. She needs to find the area of the wall, excluding the doors and the windows. A diagram of the room is shown.



- Giuliana creates a scale drawing to bring to the contractor. She uses the scale $\frac{1}{4}$ inch represents 1 foot. Copy the diagram and label the scale drawing dimensions in inches.
 - Find the area of the wall that will be covered in brick.
 - If the brick, with installation, costs \$18 per square foot, find the total cost of the job.
9. Elizabeth is a landscape designer. She created a small pond with a fountain that her company plans to manufacture and market. It is a free-form shape and the company wants to include the area of the pond on the packaging.
- She takes a diagram of the pond and places it inside a 25-ft by 25-ft square. What is the area of the square?
 - She then uses a graphing calculator to generate 5,000 random points inside the square. She finds that 3,200 of these points landed inside the pond outline. What percent of the points landed in the pond?
 - What is the area of the pond, to the nearest square foot?
10. Elizabeth (from Exercise 9) is designing another backyard pond design for her company. She follows the same procedure to find the area. She first creates an x by y rectangle to frame her pond diagram. She generates 10,000 random points inside the rectangle and p points land inside the pond. Express the area of the pond algebraically.

There's no place like home.

Judy Garland, American Actress, as Dorothy in *The Wizard of Oz*

Mortgage Application Process

8-3

Key Terms

- market value
- property tax
- real estate tax
- assessed value
- down payment
- mortgage
- fixed rate mortgage
- adjustable rate mortgage
- foreclose
- homeowner's insurance
- escrow
- front-end ratio
- back-end ratio
- debt-to-income ratio
- balloon mortgage
- interest-only mortgage

Objectives

- Compute the monthly cost of paying for a house.
- Understand the research that is necessary before you purchase a home.

WHAT DO YOU NEED TO KNOW ABOUT MORTGAGES?

Buying a house is probably the most expensive investment you will ever make. **Market value** is the amount for which a house could be sold. Homeowners pay **property taxes**, also called **real estate taxes**. The **assessed value** of a home is an amount used to determine the property taxes. The assessed value may not be the same as the market value. Property taxes help pay for government services, such as schools, libraries, and police.

After making the required **down payment**, most people take out a loan to pay the balance owed on their new home. These loans are **mortgages**. Because interest rates differ, shopping for a mortgage can be important. You should become familiar with the following mortgage vocabulary.

- **Fixed rate mortgage** A fixed rate mortgage is a mortgage in which the monthly payment and annual percentage rate (APR) remain the same throughout the entire loan period.
- **Adjustable rate mortgage** An adjustable rate mortgage is a mortgage in which the monthly payment and the APR may change, as specified in the signed agreement.
- **Foreclosure** The bank forecloses on (takes possession of) the home and sells it if the homeowner cannot pay the mortgage.
- **Homeowner's insurance** Insurance that covers damage to the home due to fire, and other natural disasters. It also covers the contents of the home in case of theft or vandalism.

Most mortgage loans are repaid over 15 to 30 years, which means a home buyer is taking on a long-term financial responsibility.

Skills and Strategies

Here you will examine what costs must be researched by a prospective home buyer before committing to the responsibility of a monthly mortgage payment for many years.

EXAMPLE 1

- Heather is planning to buy a home. She has some money for a down payment already. She sees a home she would like and computes that she would need to borrow \$190,000 from a bank over a 30-year period. The APR is 6.4%. What will be her total interest for the 30 years?



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SOLUTION Recall the monthly payment formula from Chapter 4.

$$M = \frac{p \left(\frac{r}{12} \right) \left(1 + \frac{r}{12} \right)^{12t}}{\left(1 + \frac{r}{12} \right)^{12t} - 1}$$

where M = monthly payment
 p = principal
 r = interest rate expressed as a decimal
 t = number of years

Substitute and simplify. Round to the nearest cent.

$$M = \frac{190,000 \left(\frac{0.064}{12} \right) \left(1 + \frac{0.064}{12} \right)^{12(30)}}{\left(1 + \frac{0.064}{12} \right)^{12(30)} - 1} \approx 1,188.46$$

The monthly payment is \$1,188.46.

There are 12 payments per year, so there are 360 payments over the 30 years. Multiply to find the sum of all 360 payments.

$$360(1,188.46) = 427,845.60$$

The sum of the monthly payments is \$427,845.60. To find the interest, subtract the principal from this amount.

$$427,845.60 - 190,000 = 237,845.60$$

Heather will pay \$237,845.60 in interest. This is almost a quarter of a million dollars, and it is just interest! Buying a home is an expensive proposition.

■ CHECK YOUR UNDERSTANDING

Don and Barbara Weinstein are looking for a home for which they would have to borrow p dollars. If they take out a 30-year loan with a monthly payment equal to M , express their interest I algebraically.

EXAMPLE 2

Jessica and Darryl Delaware are looking at a house, and they contacted the tax assessor to find out what the property taxes would be. In their town, the tax is based on the square footage and other features of the house. The classified ad describing their house is shown below. What is the annual property tax on their house if the town has a tax rate of 0.89%?

2-story Colonial with 2.5 bath, frpl, full basement, CAC, 30 × 30 ft deck, 3/4 acre, 600 sq ft first flr, 1500 sq ft second flr, 20 × 20 ft dormer, 12 × 21 ft garage, 16 × 32 ft vinyl pool, gas ht, excellent cond. \$289K

SOLUTION Property tax is based on the assessed value of the house. The Delaware's received a copy of how the assessed values are computed, shown at the right. Some of the assessed values are based on square footage, and some are flat rates. Compute the assessed value for each part.

Add all the assessed values to find the total assessed value of the home.

Structural Rates per Square Foot	Flat Rates
1st floor over basement \$3.00	land \$1,000 per acre
1st floor over slab \$2.25	1st bathroom \$0
2nd floor \$2.25	extra full bathroom \$100
garage \$1.00	half bathroom \$50
dormer \$1.00	fireplace \$125
barn \$0.75	tennis court \$375
deck \$0.50	spa \$125
shed \$0.70	central air conditioning \$200
vinyl-lined pool \$0.75	gas heat \$700

	Square footage	Assessed Value
first floor	600	$600 \times 3 = 1,800$
second floor	1,500	$1,500 \times 2.25 = 3,375$
dormer	$20 \times 20 = 400$	$400 \times 1 = 400$
garage	$12 \times 21 = 252$	$252 \times 1 = 252$
deck	$30 \times 30 = 900$	$900 \times 0.5 = 450$
pool	$16 \times 32 = 512$	$512 \times 0.75 = 384$
land		$\frac{3}{4} \times 1,000 = 750$
extra bathrooms		$1\frac{1}{2} = 100 + 50 = 150$
fireplace		125
central air conditioning		200
gas heat		700

$$1,800 + 3,375 + 400 + 252 + 450 + 384 + 750 + 150 + 125 + 200 + 700 = 8,586$$

The assessed value is \$8,586.

Multiply the assessed value by the tax rate to compute the annual property tax. The town has a tax rate of 89%. Find 89% of \$8,586 to find the annual property tax.

$$0.89(8,586) = 7,641.54$$

The property tax on the house is \$7,641.54 per year.

■ CHECK YOUR UNDERSTANDING

The assessed value of a home is a dollars and the tax rate, expressed as a decimal, is r . Express the property tax P algebraically.

EXAMPLE 3

Kevin and Cathy Mackin have a mortgage with National Trust Bank. The bank requires that the Mackins pay their homeowner's insurance, property taxes, and mortgage in one monthly payment to the bank.

Their monthly mortgage payment is \$1,233.56, their semi-annual property tax bill is \$5,206, and their annual homeowner's insurance bill is \$1,080. How much is the monthly payment they make to National Trust?

SOLUTION The bank wants the insurance and taxes paid monthly so the Mackins do not have large bills to pay at the end of the year. The bank holds the insurance and property tax money and pays those bills for the Mackins when they are due. This is holding money in **escrow**.

Divide the annual insurance by 12 to get a monthly amount.

$$1,080 \div 12 = 90$$

The Mackins must pay \$90 per month into escrow for their homeowner's insurance.

Divide the semi-annual property tax by 6, and round to the nearest cent to get a monthly amount for the property tax.

$$5,206 \div 6 = 867.67$$

The Mackins must pay \$867.67 per month into escrow for their property taxes.

The monthly payment to National Trust is the sum of the monthly mortgage, insurance, and taxes.

$$1,233.56 + 90 + 867.67 = 2,191.23$$

The Mackins pay the bank \$2,191.23 each month.

■ CHECK YOUR UNDERSTANDING

Michelle and Dan Zlotnick pay their mortgage, insurance, and property taxes in one monthly payment to the bank. If their monthly mortgage payment is m dollars, their annual property tax payment is p dollars, and their quarterly homeowner's insurance payment is h dollars, express the amount they pay the bank monthly algebraically.



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EXAMPLE 4

Tom and Lori Courtney are considering buying a house and are researching the potential costs. Their adjusted gross income is \$135,511. The monthly mortgage payment for the house they want would be \$1,233. The annual property taxes would be \$9,400, and the homeowner's insurance premium would cost them \$876 per year. Will the bank lend them \$190,000 to purchase the house?

SOLUTION Banks use several factors, including credit rating, to decide if they will lend money. The bank wants to be paid back. They want assurance that the borrowers can afford the monthly payments.

One indicator is the **front-end ratio**, which is a ratio of monthly housing expenses to monthly gross income.

$$\text{Front-end ratio} = \frac{\text{Monthly housing expenses}}{\text{Monthly gross income}}$$

Banks often want the front-end ratio to be 28% or less before they lend the money.

Find the monthly amount for property tax.

$$\text{Annual property tax} \div 12 = \frac{9,400}{12} \approx 783.33$$

Rounded to the nearest cent, the monthly property tax is \$783.33.

Find the monthly amount for homeowner's insurance.

$$\text{Annual insurance} \div 12 = \frac{876}{12} = 73$$

Find the monthly gross income.

$$\text{Annual gross income} \div 12 = \frac{135,511}{12} \approx 11,292.58$$

The monthly gross income is \$11,292.58, rounded to the nearest cent.

Substitute these values into the front-end ratio. Convert the decimal equivalent to a percent.

$$\text{Front-end ratio} = \frac{1,233 + 783.33 + 73}{11,292.58} = \frac{2,089.33}{11,292.58} \approx 0.185 = 18.5\%$$

The front-end ratio is 18.5%.

The Courtneys' front-end ratio is less than 28%, so the bank would say they can afford the mortgage on this house based on the front-end ratio.

■ CHECK YOUR UNDERSTANDING

Ken and Julie Frederick have an adjusted gross income of x dollars. They are looking at a new house. Their monthly mortgage payment would be m dollars. Their annual property taxes would be p dollars, and their annual homeowner's premium would be h dollars. Express their front-end ratio algebraically.

EXAMPLE 5

Bill and Terry Noke are considering buying a house and need to figure out what they can afford and what a bank will lend them. Their adjusted gross income is \$166,988. Their monthly mortgage payment for the house they want would be \$1,544. Their annual property taxes would be \$9,888, and the homeowner's insurance premium would cost them \$1,007 per year. They have a \$510 per month car loan, and their average monthly credit card bill is \$5,100. Would the bank lend them \$210,000 to purchase their house?

SOLUTION In Example 4 you learned about the front-end ratio that banks use to assess potential borrowers. Banks also use the **back-end ratio**, or **debt-to-income ratio**, which takes into account a borrower's regular monthly debts, such as car loans, alimony, child support, and credit card bills.



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$$\text{Back-end ratio} = \frac{\text{Total monthly expenses}}{\text{Monthly gross income}}$$

Banks generally want a back-end ratio less than 36% to approve a mortgage application.

Find the monthly amounts for the homeowner's insurance and the property taxes to the nearest dollar.

$$\text{Monthly homeowner's insurance} \quad 1,007 \div 12 \approx 84$$

$$\text{Monthly property tax} \quad 9,888 \div 12 = 824$$

The total monthly expenses are the sum of the mortgage payment, property tax, homeowners insurance, car payment, and credit card payments.

$$\text{Add.} \quad 1,544 + 824 + 84 + 510 + 5,100 = 8,062$$

Find the Nokes' monthly gross income by dividing by 12 and rounding to the nearest dollar.

$$166,988 \div 12 \approx 13,916$$

Substitute these values into the back-end ratio.

$$\text{Back-end ratio} = \frac{8,062}{13,916} \approx 0.579$$

The back-end ratio for the Nokes is 58%.

The back-end ratio for the Nokes is greater than 36%. The bank will not give them a loan for \$210,000.

■ CHECK YOUR UNDERSTANDING

Find the back-end ratio to the nearest percent for the Nokes in Example 5, if they pay off their car, and Terry gets a \$12,000 raise.

EXAMPLE 6

Chris and Scott Halloran are opening a new restaurant. They take out a 6.1%, 15-year, \$300,000 mortgage on the building, but they do not have a lot of money because they are spending what they have to get the business started. Years in the future they intend to have much more money from the success of the restaurant. Can they get a loan that will fit well with their current and future incomes? How much will they pay in interest for the loan? What are the monthly payments?

SOLUTION The Hallorans can take out a balloon mortgage. A **balloon mortgage** features a very high last payment, with all other payments being relatively low.

One type of balloon loan is an **interest-only** balloon mortgage where only the interest is paid until the final month.

Use the monthly payment formula. Substitute, simplify, and round to the nearest cent.

$$M = \frac{300,000 \left(\frac{0.061}{12} \right) \left(1 + \frac{0.061}{12} \right)^{12(15)}}{\left(1 + \frac{0.061}{12} \right)^{12(15)} - 1} \approx 2,547.81$$

To find the interest due on the Halloran's loan, first find the total amount due for the loan. Then subtract the \$300,000 principal from the total paid.

The loan is for 15 years, so there will be $12 \times 15 = 180$ payments.

Payment amount \times 180 $2,547.81 \times 180 = 458,605.80$

Total paid – principal $458,605.80 - 300,000 = 158,605.80$

The interest on the Halloran's loan, rounded to the nearest dollar, is \$158,606.

If the last payment is the \$300,000 balloon, the first 179 payments equal the interest only.

Divide. $158,606 \div 179 \approx 886.07$

The first 179 monthly payments are \$886.07, and the 180th (last) payment, at the end of the 15 years, is the balloon payment of \$300,000.

Notice that the initial monthly payments were low, allowing the Hallorans to put money into their business.

Keep in mind that they had to start saving for the balloon payment years in advance since it is so high.

There are other ways to set up balloon payments, but all of them feature the large final payment.

■ CHECK YOUR UNDERSTANDING

The total interest on a 20-year balloon mortgage with principal p dollars is x dollars. If just the interest is paid before the final balloon payment, express the monthly payment before the balloon payment amount algebraically.

Applications

There's no place like home.

Judy Garland, American Actress, as Dorothy in *The Wizard of Oz*

1. Interpret the quote in the context of your own home.
2. The Smiths took out a \$130,000, 30-year mortgage at an APR of 6.5%. The monthly payment was \$821.69. What will be their total interest charges after 30 years?
3. If you borrow \$120,000 at an APR of 7% for 25 years, you will pay \$848.13 per month. If you borrow the same amount at the same APR for 30 years, you will pay \$798.36 per month.
 - a. What is the total interest paid on the 25-year mortgage?
 - b. What is the total interest paid on the 30-year mortgage?
 - c. How much more interest is paid on the 30-year loan? Round to the nearest dollar.
 - d. If you can afford the difference in monthly payments, you can take out the 25-year loan and save all the interest from part c. What is the difference between the monthly payments of the two different loans? Round to the nearest dollar.
4. United Bank offers a 15-year mortgage at an APR of 6.2%. Capitol Bank offers a 25-year mortgage at an APR of 6.5%. Marcy wants to borrow \$120,000.
 - a. What would the monthly payment be from United Bank?
 - b. What would the total interest be from United Bank? Round to the nearest ten dollars.
 - c. What would the monthly payment be from Capitol Bank?
 - d. What would the total interest be from Capitol Bank? Round to the nearest ten dollars.
 - e. Which bank has the lower total interest, and by how much?
 - f. What is the difference in the monthly payments?
 - g. How many years of payments do you avoid if you decide to take out the shorter mortgage?
5. The assessed value of the Weber family's house is \$186,000. The annual property tax rate is 2.15% of assessed value. What is the property tax on the Weber's home?
6. The monthly payment on a mortgage with a principal of p dollars is m dollars. The mortgage is taken out for y years. Express the interest I as a function of p , m , and y .
7. The market value of Christine and Gene's home is \$275,000. The assessed value is \$230,000. The annual property tax rate is \$17.50 per \$1,000 of assessed value.
 - a. What is the property tax on their home?
 - b. How much do they pay monthly toward property taxes? Round your answer to the nearest cent.

8. Jim is taking out a \$135,000 mortgage. His bank offers him an APR of 6.25%. He wants to compare monthly payments on a 20- and a 30-year loan. Find, to the nearest dollar, the difference in the monthly payments for these two loans.
9. The Joseph family took out a \$175,000, 25-year mortgage at an APR of 6%. The assessed value of their house is \$9,000. The annual property tax rate is 97.22% of assessed value. What is the annual property tax?
10. The Jordans are considering buying a house with a market value of \$250,000. The assessed value of the house is a dollars. The annual property tax is \$2.45 per \$100 of assessed value. What is the property tax on this house?
11. Allison has a mortgage with North End Bank. The bank requires that she pay her homeowner's insurance, property taxes, and mortgage in one monthly payment. Her monthly mortgage payment is \$1,390, her semi-annual property tax bill is \$3,222, and her quarterly homeowner's bill is \$282. How much does Allison pay North End Bank each month?
12. Mike and Cheryl had an adjusted gross income of a dollars. Mike just got a \$3K raise and Cheryl got a \$1.5K raise. They are considering moving to a new house with monthly mortgage payments of m dollars, annual property taxes of p dollars, and annual homeowner's premium of h dollars. Express their front-end ratio algebraically.
13. The Ungers have an adjusted gross income of \$117,445. They are looking at a new house that would carry a monthly mortgage payment of \$1,877. Their annual property taxes would be \$6,780, and their semi-annual homeowner's premium would be \$710.
 - a. Find their front-end ratio to the nearest percent.
 - b. Assume that their credit rating is good. Based on the front-end ratio, would the bank offer them a loan? Explain.
 - c. The Ungers have a monthly car loan of \$430, and their average monthly credit card bill is \$5,100. Mr. Unger is also paying \$1,000 per month in child support from a previous marriage. Compute the back-end ratio to the nearest percent.
 - d. If the bank used both the front-end and back-end ratios to decide on mortgage approval, would the Ungers get their mortgage? Explain.
14. Andy is a single father who wants to purchase a home. His adjusted gross income for the year is a dollars. His monthly mortgage is m dollars, and his annual property tax bill is p dollars. His monthly credit card bill is c dollars, and he has a monthly car loan for d dollars. His quarterly homeowner's bill is h dollars. Express Andy's back-end ratio algebraically.
15. Ron has a homeowner's insurance policy, which covers theft, with a deductible of d dollars. Two bicycles, worth b dollars each, and some tools, worth t dollars, were stolen from his garage. If the value of the stolen items was greater than the deductible, represent the amount of money the insurance company will pay algebraically.
16. Find the monthly payment (before the balloon payment) for a 20-year, interest-only balloon mortgage for \$275,000 at an APR of 8%. Round to the nearest ten dollars.

New Listing: Cape-Cod style home w/ 2 baths, 700 sq. ft first flr., upstairs 15 × 26 dormer, 12 × 20 garage, gas heat, frpl, basement. Property 1/2 acre plot w/ 5 × 12 shed and tennis court.
\$301K

17. Siegell's Locksmith Shop is taking out a mortgage on a new building. It is going to be an interest-only, 12-year balloon mortgage for \$350,000. The APR is 7.1%. The last payment will be the balloon payment of the full principal.
 - a. Find the total interest for the 12-year mortgage.
 - b. Find the total number of monthly payments, not including the final balloon payment.
 - c. Find the amount of each monthly payment if the payments are interest-only. Round to the nearest cent.
 - d. Find the difference between the regular monthly payment and the balloon payment, to the nearest hundred dollars.
 - e. If the mortgage was not a balloon mortgage, what would be the amount of the monthly payment, rounded to the nearest cent?
18. An interest-only balloon mortgage for a principal of p dollars for 18 years has total interest of t dollars. Express the amount of each monthly payment before the balloon payment algebraically.
19. Using the table from Example 2, find the assessed value of the house in this classified ad.
20. Mark and Beth are looking at four different homes. They created this spreadsheet to estimate escrow calculations more easily. They will pay the property tax and homeowner's insurance each month with their mortgage payment. The bank will hold these two amounts in escrow until those bills need to be paid, which is every six months. Each line represents data for a different home they are looking at. Mark and Beth input values for the mortgage, property tax, and homeowner's insurance in row 2, columns A, B, and C.

	A	B	C	D	E
	Monthly Mortgage	Annual Property Tax	Annual Homeowner's Insurance	Monthly Escrow Payment	Escrow Balance with Interest after Six Months
1					
2	1,435	5,900	1,234	c.	d.
3	1,987	8,766	1,567	e.	f.
4	2,081	8,944	1,540	g.	h.
5	1,873	7,711	1,564	i.	j.

Future Value of a Periodic Deposit

$$B = \frac{P \left(\left(1 + \frac{r}{n} \right)^{nt} - 1 \right)}{\frac{r}{n}}$$

where B = balance at the end of the six months
 P = periodic deposit amount, which is the monthly escrow
 r = annual interest rate expressed as a decimal
 n = number of times the interest is compounded annually
 t = length of the investment in years

- a. Write the spreadsheet formula for cell D2 that will compute the escrow balance after six months.
If the monthly escrow payments get 1% interest compounded monthly, Mark and Beth can compute the value of the escrow account in six months. Look at this as finding the future value of a periodic deposit. Recall the formula from Lesson 3-8 shown at the left.
- b. Write the spreadsheet formula for cell E2 that will compute the escrow balance after six months, with the given interest rate and monthly compounding.
- c-j. Fill in the missing entries.

Owning a home is a keystone of wealth . . . both financial affluence and emotional security.

Suze Orman, Author, TV Personality, and Personal Finance Expert

Purchase a Home

8-4

Key Terms

- recurring costs
- non-recurring costs
- closing
- closing costs
- earnest money deposit
- attorney fee
- origination fee
- title
- title search
- points
- origination points
- discount points
- prepaid interest
- arrears
- transfer tax
- amortization table
- initial rate
- adjustment period
- hybrid ARM

Objectives

- Estimate closing costs.
- Create an amortization table for a fixed rate mortgage.
- Create an amortization table for a fixed rate mortgage with extra payments.
- Investigate the amortization table for an adjustable rate mortgage.

WHAT WILL THE AMERICAN DREAM COST YOU?

Once you have your mortgage approval, you are a big step closer to home ownership. Below are a few questions that you must investigate thoroughly before buying a home

- What is the cost of the home?
- Will you need to make a down payment?
- Where is the home located?
- How many rooms does the home have?
- What is the size of the property you will own?
- What condition is the house and property in?
- What type of heating/cooling system does the house have?
- What is the approximate cost of running the house (electricity, gas, water, and so on)?

One of the biggest concerns for a prospective homeowner is the costs in both the immediate and the distant future. These costs are in two categories: recurring costs and non-recurring costs. **Recurring costs** are costs that occur on a regular basis. Some examples of recurring costs are mortgage payments, insurance payments, and property taxes.

Non-recurring costs are one-time costs. Moving costs and many of the costs at the closing are non-recurring. The **closing** is a meeting attended by the buyer, seller, their attorneys, and a representative of the lending institution. The official sale takes place at this meeting. The buyer is responsible for paying **closing costs**. Although they can differ from state to state, some of the most common non-recurring closing costs are listed and explained on the next page.



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- **Earnest Money Deposit** The earnest money, or good-faith deposit, is the money paid to the seller by an interested buyer to show that the buyer is serious about buying the house.
- **Attorney Fees** These are fees paid to the attorney in return for representation at the closing.
- **Origination Fee** This fee is money paid to the lending institution for the paperwork involved in the loan application process.
- **Title** The title is the legal claim of property ownership. It is common practice that before property can change hands, a **title search** is conducted. A title search is a procedure used to make sure that the seller does actually hold title to the property being sold.
- **Points** Points are extra fees charged by the lending institution for the use of their money. Each point is equivalent to 1% of the loan amount. There are two types of points. **Origination points** are similar to origination fees. They are collected from the buyer as a means of paying for the loan application process. **Discount points** are points that reduce the interest rate of the loan. They generally lower the interest rate about 0.25% on a fixed rate mortgage and 0.375% on an adjustable rate mortgage. These percentages vary depending upon the lending institution.
- **Prepaid Interest** Mortgage interest is not paid like rent. Rent is paid ahead of the time you live in the home. Mortgage interest is paid in **arrears**. Interest starts accruing (building) at the beginning of each month and continues throughout the month. When you make your mortgage payment, you are paying the interest that has built up on money borrowed during the month that just passed. Prepaid interest at the closing is the amount of mortgage interest due to cover the time from the closing date to when the first mortgage payment is due. For example, if you close on the 10th day of a 30-day month, you will need to prepay 20 days of interest at the closing.
- **Transfer Tax** This is a fee that is charged for the transfer of title from the seller to the buyer.

Skills and Strategies

Here you will learn about the financial aspects of the closing and the years that follow the purchase of a home.

EXAMPLE 1

Leah and Josh are buying a \$600,000 home. They have been approved for a 7.25% APR mortgage. They made a 15% down payment and will be closing on September 6th. How much should they expect to pay in prepaid interest at the closing?

SOLUTION First determine the amount that Josh and Leah borrowed. Since they made a 15% down payment, multiply \$600,000 by 0.15. Then subtract the down payment from the original amount.

$$\text{Down payment} \quad 600,000 \times 0.15 = 90,000$$

$$\text{Loan amount} \quad 600,000 - 90,000 = 510,000$$

You can also calculate the loan amount by recognizing that 15% of the purchase price was the down payment. Therefore, subtract 15% from 100% to find the percent of the purchase price that is the loan amount.

$$100\% - 15\% = 85\%$$

Then multiply the purchase price by the percent that remains to be financed by the mortgage to find the loan amount.

$$600,000 \times 0.85 = 510,000$$

Josh and Leah's first mortgage payment will be due on October 1. At that time they will prepay interest from September 7 to September 30 for a total of 24 days. The amount of prepaid interest is calculated as follows.

Determine the annual interest by multiplying the APR times the amount borrowed.

$$510,000 \times 0.0725 = 36,975$$

There is \$36,975 in annual interest for this loan.

Determine the daily amount of interest due by dividing the annual interest by 365 calendar days.

$$36,975 \div 365 \approx \$101.30$$

It will cost \$101.30 in interest per day.

Multiply the daily interest rate by the number of days to be paid in arrears.

$$101.30 \times 24 = 2,431.20$$

Leah and Josh will owe \$2,431.20 in prepaid interest for the remainder of September.

■ CHECK YOUR UNDERSTANDING

How much will be charged in prepaid interest on a \$400,000 loan with an APR of 6% that was closed on December 17?

EXAMPLE 2

Leah and Josh know that they will have to bring their checkbook to the closing. What might they expect to pay in total at the closing?

SOLUTION Although there are no guarantees about what they will pay, the rule of thumb is that they can expect the closing costs to run from 2% to 7% of the purchase price. These numbers vary depending upon the location of the house and on any special circumstances.

$$600,000 \times 0.02 = 12,000$$

$$600,000 \times 0.07 = 42,000$$

Leah and Josh should be prepared to write checks that will total from \$12,000 to \$42,000 at the closing.

■ CHECK YOUR UNDERSTANDING

Shannon had to make a down payment of 15% of the selling price of her house. She was approved for a \$340,000 mortgage. What range of costs might she expect to pay at the closing?

EXAMPLE 3

Trudy and Tom have been approved for a \$300,000, 15-year mortgage with an APR of 5.75%. How much of their first monthly payment will go to interest and principal?

SOLUTION The amount allocated to principal and interest changes from month to month. At the beginning of the loan, the interest payment is high and the principal payment is lower. Towards the end of the loan the amount the principal payment becomes larger than the interest payment. To calculate each amount, you need to determine the monthly payment. You will use a slightly altered loan formula that allows you to enter the rate as a percent rather than as an equivalent decimal. Since the decimal is needed in the calculation, the monthly rate of $\frac{r}{12}$ is divided by 100.

$$\frac{r}{12} \div 100 = \frac{r}{12} \times \frac{1}{100} = \frac{r}{1,200}$$

The new monthly payment formula is

$$M = \frac{p \left(\frac{r}{1,200} \right) \left(1 + \frac{r}{1,200} \right)^{12t}}{\left(1 + \frac{r}{1,200} \right)^{12t} - 1} \quad \text{where } \begin{array}{l} M = \text{monthly payment} \\ p = \text{principal} \\ r = \text{interest rate expressed} \\ \quad \text{as a percent} \\ t = \text{length of loan in years} \end{array}$$

Substitute and simplify.

$$M = \frac{300,000 \left(\frac{5.75}{1,200} \right) \left(1 + \frac{5.75}{1,200} \right)^{12(15)}}{\left(1 + \frac{5.75}{1,200} \right)^{12(15)} - 1} = 2491.23$$

The monthly payment on this loan is \$2,491.23.

The monthly interest can be determined using the monthly interest formula.

$$I = p \times \frac{r}{1,200} \quad \text{where } \begin{array}{l} I = \text{interest} \\ p = \text{principal} \\ r = \text{interest rate expressed as a percent} \end{array}$$

Substitute. $I = 300,000 \times \frac{5.75}{1,200} = 1,437.50$

The first monthly interest amount is \$1,437.50.

Subtract that amount from the monthly payment to get the amount paid towards the principal.

$$2,491.23 - 1,437.50 = 1,053.73$$

The amount paid towards the principal is \$1,053.73.

■ CHECK YOUR UNDERSTANDING

What percent of the monthly payment went to principal and what percent went to interest?

EXAMPLE 4

How can Trudy and Tom get an accounting of where their monthly payments will go for the first year of their mortgage?

SOLUTION In Example 3, you calculated the principal and interest for a single month. Here, Trudy and Tom need data over the course of 12 months. To determine the principal and interest amounts for an extended period of time, they should review an amortization table for their loan. An **amortization table** is a listing of the unpaid principal, the monthly payment, the amount allocated to paying down the principal, and the amount allocated to interest. There are many websites that offer mortgage amortization calculators. Some of them generate the amortization table.

Trudy and Tom can set up their own spreadsheet to generate the amortization table. They first set up rows 1–4 of the spreadsheet where the user will input the necessary data. Here the information is entered into the cells in column B in rows 1–4.

	A	B	C	D	E	F
1	Principal	300,000				
2	Interest rate as a percent	5.75				
3	Length of loan	15				
4	Number of yearly payments	12				
5	Payment Number	Beginning Balance	Monthly Payment	Towards Interest	Towards Principal	Ending Balance

Next, determine the information that they will need in the amortization table. It should contain the payment number, the beginning balance, the monthly payment, the amounts allocated towards principal and interest, and the ending balance as shown in row 5.

- Row 6 contains the formulas needed to generate the table.
- A6 1 Begin with number 1.
- B6 =B1 The beginning balance is the principal.
- C6 =(B1*(B2/1200)*(1+B2/1200)^(B4*B3))/((1+B2/1200)^(B4*B3)-1)
Monthly payment formula from Example 3.
- D6 =B6*B2/1200 This is the interest formula.
- E6 =C6-D6 The monthly payment less the interest.
- F6 =B6-E6 The ending balance is the beginning balance minus the amount towards principal.

	A	B	C	D	E	F
1	Principal	300,000				
2	Interest rate as a percent	5.75				
3	Length of loan	15				
4	Number of yearly payments	12				
5	Payment Number	Beginning Balance	Monthly Payment	Towards Interest	Towards Principal	Ending Balance
6	1	300,000.00	2,491.23	1,437.50	1,053.73	298,946.27
7	2	298,946.27	2,491.23	1,432.45	1,058.78	297,887.49
8	3	297,887.49	2,491.23	1,427.38	1,063.85	296,823.64
9	4	296,823.64	2,491.23	1,422.28	1,068.95	295,754.69
10	5	295,754.69	2,491.23	1,417.16	1,074.07	294,680.61
11	6	294,680.61	2,491.23	1,412.01	1,079.22	293,601.40
12	7	293,601.40	2,491.23	1,406.84	1,084.39	292,517.01
13	8	292,517.01	2,491.23	1,401.64	1,089.59	291,427.42
14	9	291,427.42	2,491.23	1,396.42	1,094.81	290,332.61
15	10	290,332.61	2,491.23	1,391.18	1,100.05	289,232.56
16	11	289,232.56	2,491.23	1,385.91	1,105.32	288,127.23
17	12	288,127.23	2,491.23	1,380.61	1,110.62	287,016.61

- Set up row 7 so it can be copied into subsequent rows (filled down).
- When copying formulas, the spreadsheet advances the cell address in the formula down by one row. If you add \$ to the cell address, it won't change when copying. The entries in row 9 should be as follows.
- A7 =A6+1 This will add 1 to each payment number.
- B7 =F6 The beginning balance is last month's ending balance.
- C7 =\$C\$6 Put \$ in the cell address to keep it from changing.
- D7 =B7*\$B\$2/1200 Again add \$ to keep cell address fixed.
- E7 =C7-D7
- F7 =B7-E7
- The completed table for the first 12 payments is shown above.

■ CHECK YOUR UNDERSTANDING

Adding a sum cell to the bottom of the *monthly payment, towards interest, and towards principal* columns yields the following totals at the end of the first year of payments.

Payments for 12 Months: \$29,894.76
 Interest for 12 Months: \$16,911.38
 Principal for 12 Months: \$12,983.39

At the end of the 12-month period, what percent of the principal has been paid off?

EXAMPLE 5

Trudy and Tom decide to make an extra payment of \$100 each month to reduce their principal. They adjust their spreadsheet as shown. What formula change(s) did they make in row 6 so that the extra payment could be accounted for?

	A	B	C	D	E	F	G
1	Principal	300,000					
2	Interest rate as a percent	5.75					
3	Length of loan	15					
4	Number of yearly payments	12					
5	Payment Number	Beginning Balance	Monthly Payment	Extra Payment	Towards Interest	Towards Principal	Ending Balance
6	1	300,000.00	2,491.23	100.00	1,437.50	1153.73	298,846.27
7	2	298,846.27	2,491.23	100.00	1,431.97	1159.26	297,687.01
8	3	297,687.01	2,491.23	100.00	1,426.42	1164.81	296,522.20
9	4	296,522.20	2,491.23	100.00	1,420.84	1170.39	295,351.80
10	5	295,351.80	2,491.23	100.00	1,415.23	1176.00	294,175.80
11	6	294,175.80	2,491.23	100.00	1,409.59	1181.64	292,994.16
12	7	292,994.16	2,491.23	100.00	1,403.93	1187.30	291,806.86
13	8	291,806.86	2,491.23	100.00	1,398.24	1192.99	290,613.87
14	9	290,613.87	2,491.23	100.00	1,392.52	1198.71	289,415.17
15	10	289,415.17	2,491.23	100.00	1,386.78	1204.45	288,210.72
16	11	288,210.72	2,491.23	100.00	1,381.01	1210.22	287,000.50
17	12	287,000.50	2,491.23	100.00	1,375.21	1216.02	285,784.48

SOLUTION There is a new column D, so the formulas need to be adjusted in the following columns.

$$E6 = B6 * B2 / 1200$$

$$F6 = C6 + D6 - E6$$

The monthly payment plus the extra payment less the monthly interest is the amount that goes towards the principal.

$$G6 = C6 - F6$$

■ CHECK YOUR UNDERSTANDING

Examine the loan summaries below for each of the two situations outlined above. How much interest and loan time was saved by making the extra \$100 in payments toward principal each month?

	Without Extra Payment	With Extra Payment
Monthly Payment	2,491.23	2,491.23
Scheduled Payments	180	180
Actual Payments	180	170
Total Extra Payments	0	16,900.00
Total Interest	148,421.45	138,610.08

Adjustable Rate Mortgages (ARMs)

In the previous examples, each of the homebuyers had a fixed rate mortgage. In a fixed rate mortgage, the interest rate remains the same throughout the term of the loan. In an *adjustable rate mortgage* or *ARM*, the interest rate can change periodically. Therefore, the monthly payments change as well, based upon those rates.

Here is how an adjustable mortgage works. Lenders quote you an **initial rate** that stays in effect for an agreed upon period of time. This can be as short as 1 month to several years. The monthly payment is based upon that initial rate. Often, the initial interest rate quoted is tied into a customer's credit worthiness. In an ARM, the interest rate and monthly payment will change periodically. The period between rate changes is known as the **adjustment period**. A loan with a 1-year adjustment period is known as a 1-year ARM. Here, the interest rate and the monthly payment may change at the end of one year's adjustment period.



Some ARMS, known as **hybrid ARMs**, are a combination of a fixed rate period of time with an adjustable rate period of time. A 3/1 hybrid ARM indicates that the initial interest rate is fixed for the first 3 years and then there is an adjustment period every year thereafter for the life of the loan. There are many types of adjustable mortgages and many different rules and regulations attached to those mortgages. This is just another example of “Buyer Beware!” You must always be sure to read the fine print.

EXAMPLE 6

Chris and Gene have a 6-month adjustable 15-year mortgage. They borrowed \$300,000 and were quoted an initial rate of 5%. After 6 months, their rate increased by 1%. Examine the following spreadsheet for the first year of payments. How were the amounts for payment 7 calculated?

	A	B	C	D	E	F	G
1	Principal	300,000					
2	Interest rate as a percent	5	6				
3	Length of loan	15	14.5				
4	Number of yearly payments	12					
5	Payment Number	Beginning Balance	Monthly Payment	Towards Interest	Towards Principal	Ending Balance	Interest Rate
6	1	300,000.00	2,372.38	1,250.00	1,122.38	298,877.62	5%
7	2	298,877.62	2,372.38	1,245.32	1,127.06	297,750.56	5%
8	3	297,750.56	2,372.38	1,240.63	1,131.75	296,618.81	5%
9	4	296,618.81	2,372.38	1,235.91	1,136.47	295,482.34	5%
10	5	295,482.34	2,372.38	1,231.18	1,141.20	294,341.13	5%
11	6	294,341.13	2,372.38	1,226.42	1,145.96	293,195.17	5%
12	7	293,195.17	2,526.94	1,465.98	1,060.96	292,134.21	6%
13	8	292,134.21	2,526.94	1,460.67	1,066.27	291,067.94	6%
14	9	291,067.94	2,526.94	1,455.34	1,071.60	289,996.34	6%
15	10	289,996.35	2,526.94	1,449.98	1,076.96	288,919.39	6%
16	11	288,919.39	2,526.94	1,444.60	1,082.34	287,837.05	6%
17	12	287,837.05	2,526.94	1,439.19	1,087.75	286,749.30	6%

SOLUTION In the spreadsheet you need to adjust the formulas for payments 7–12. Enter an interest rate of 6% in cell C2 and a length of loan of 14.5 years in cell C3. Adjust the formulas in row 12 so they use the adjusted interest rate and length of loan. Then copy those formulas into rows 13 to 17.

The new ending balance is \$286,749.30.

■ CHECK YOUR UNDERSTANDING

How much of a difference did the 1% adjustment in interest rate make in the monthly payment and the amounts towards interest and principal?

Applications

Owning a home is a keystone of wealth . . . both financial affluence and emotional security.

Suze Orman, Author, TV Personality, and Personal Finance Expert

1. Explain how the quote can be interpreted.
2. Del is buying a \$250,000 home. He has been approved for a 5.75% mortgage. He was required to make a 15% down payment and will be closing on the house on July 15. How much should he expect to pay in prepaid interest at the closing?
3. Bonnie is purchasing an apartment for \$180,000. She has been approved for a 6% mortgage. She put 10% down and will be closing on April 22. How much should she expect to pay in prepaid interest?

	A	B
1	Enter the loan amount.	
3	Enter the closing date.	
5	Enter number of days in month.	
7	Enter the APR for the loan.	
9	Interest due for one year.	
10	Daily interest due.	
11	Interest due from closing date until the end of the month.	

4. This spreadsheet can be used to calculate the amount of prepaid interest a buyer will need to pay at the closing. Write formulas for cells B9, B10, and B11.
5. Jason is closing on a \$430,000 home. He made a 13% down payment and is borrowing the rest. What is the approximate range of costs that he might expect to pay at the closing?
6. Becky was told that based on the price of her home, her approximate closing costs would range from \$4,000 to \$14,000. How much was the price of her home?

7. Celine and Don have been approved for a \$400,000, 20-year mortgage with an APR of 6.55%. Using the mortgage and interest formulas, set up a two-month amortization table with the headings shown and complete the table for the first two months.

Payment Number	Beginning Balance	Monthly Payments	Towards Interest	Towards Principal	Ending Balance
----------------	-------------------	------------------	------------------	-------------------	----------------

8. Rob has been approved for a \$275,000, 15-year mortgage with an APR of 5.9%. Using the mortgage and interest formulas, set up a table with the above headings and complete the table for the first two months.
9. Use a spreadsheet to generate the first year of payments in a loan amortization table for a \$200,000, 10-year mortgage with an APR of 7%.
10. Use a spreadsheet to generate the last year of payments in a loan amortization table for a \$600,000, 15-year mortgage with an APR of 5.5%
11. Shannon took out a \$300,000, 15-year mortgage with an APR of 7%. The first month she made an extra payment of \$400. What was her ending balance at the end of that first month?

12. Examine the loan amortization table for the last 5 months of a \$500,000, 15-year mortgage with an APR of 5.75%. Determine the missing table amounts.

Payment Number	Beginning Balance	Monthly Payment	Towards Interest	Towards Principal	Ending Balance
176	20,465.13	a.	98.06	4,053.99	16,411.14
177	b.	a.	78.64	4,073.41	12,337.73
178	12,337.73	a.	c.	4,092.93	8,244.79
179	8,244.79	a.	39.51	d.	4,132.25
180	4,132.25	a.	19.80	4,132.25	e.

13. Examine the loan amortization table for a \$210,000, 15-year mortgage with an APR of 6%. The borrower paid an extra \$100 each month towards the principal. Determine the missing amounts.

Payment Number	Beginning Balance	Monthly Payment	Extra Payment	Towards Interest	Towards Principal	Ending Balance
1	210,000.00	a.	100.00	1,050.00	822.10	209,177.90
2	b.	a.	100.00	1,045.89	826.21	c.
3	208,351.69	a.	100.00	d.	830.34	207,521.35
4	207,521.35	a.	100.00	1,037.61	e.	206,686.86
5	206,686.86	a.	100.00	1,033.43	838.67	205,848.19

14. Examine this portion of an amortization table for an adjustable rate mortgage that had a 1-year initial rate period of 4.25% and increased to 5.25% after that period ended. Determine the missing amounts.

Interest Rate	Payment Number	Beginning Balance	Monthly Payment	Towards Interest	Towards Principal	Ending Balance
4.25%	11	204,344.02	a.	723.72	576.67	b.
4.25%	12	b.	a.	721.68	578.71	203,188.63
5.25%	13	203,188.63	c.	888.95	521.19	202,667.44
5.25%	14	202,667.44	c.	d.	523.47	202,143.96
5.25%	15	202,143.96	c.	884.38	e.	201,618.20

15. Tom took out a \$440,000 15-year adjustable rate mortgage with a 4% initial 6-month rate. The amortization table for the initial rate period is shown. After the first 6 months, the rate went up to 5%. Calculate the next line of the table.

Payment Number	Beginning Balance	Monthly Payment	Toward Interest	Toward Principal	Ending Balance
1	440,000.00	3,254.63	1,466.67	1,787.96	438,212.04
2	438,212.04	3,254.63	1,460.71	1,793.92	436,418.12
3	436,418.12	3,254.63	1,454.73	1,799.90	434,618.22
4	434,618.22	3,254.63	1,448.73	1,805.90	432,812.32
5	432,812.32	3,254.63	1,442.71	1,811.92	431,000.40
6	431,000.40	3,254.63	1,436.67	1,817.96	429,182.44

Home is a place you grow up wanting to leave, and grow old wanting to get back to.

John Ed Pearce, Journalist

8-5

Rentals, Condominiums, and Cooperatives

Objectives

- Compute costs of purchasing a cooperative or a condominium.
- Understand the advantages and disadvantages of different forms of homes.

Key Terms

- condominium
- maintenance fee
- co-op apartment
- cooperative
- landominium
- board of directors
- equity

WHAT ALTERNATIVES ARE THERE TO PURCHASING A SINGLE-FAMILY HOME?

Maintaining your own house requires time as well as money. Mowing lawns, shoveling snow, and making repairs keep the average homeowner very busy. Some people prefer not to be responsible for these chores.

As discussed in Lesson 8-1, you can rent an apartment. Apartments have rules regarding pets, noise, sanitation, and physical appearance that all tenants must follow. Since tenants are not buying the home, they do not have to make a large down payment. They make monthly rental payments. Since tenants do not own the apartment that they occupy, they cannot deduct property taxes or mortgage interest on their income tax forms, and they can't make a profit from the sale of their homes.

A **condominium** is a form of home ownership where each unit is individually owned. The common parts of the property, such as the grounds, are jointly owned. Condominium owners are responsible for

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the maintenance of the inside of their own units. Condominium owners are charged a **maintenance fee**, which is used to hire workers to maintain common areas, such as lawns, outside walls, decks, roofs, sidewalks, and roads. Like single-family houses, condominiums can be purchased and sold. There are deeds, closing costs, property taxes, points, mortgage payments, and so on.

A **co-op apartment** or **cooperative** is another form of homeownership. A cooperative is a corporation that owns a group of apartments. The corporation takes out a mortgage to buy the entire apartment complex. Investors purchase shares in the co-op, and these shares allow them to occupy the apartments. Co-op owners do not own their individual apartment. They own a portion of the entire cooperative development. They can sell their shares of ownership and keep any profit from the sale. Co-op owners pay a monthly maintenance fee that covers their share of the maintenance of the apartment complex. Part of this fee covers the payment that the cooperative corporation must make toward the mortgage loan each month.

Condominiums and cooperatives usually have a **board of directors** elected by the homeowners to manage business matters. Condo and co-op owners may vote on major issues.

There are also **landominiums** where the owner owns both the home and the land on which the home is built. As with a condominium, a homeowner's association provides landscaping, maintenance, and other services and amenities such as swimming pools and tennis courts.

There are advantages and disadvantages that characterize all types of independent living. You must decide what type best suits your lifestyle and financial situation.

Skills and Strategies

EXAMPLE 1

Last year, Burt paid a monthly condominium maintenance fee of \$912. Fifteen percent of this fee covered his monthly property taxes. How much did Burt pay last year in property taxes on his condo?

SOLUTION Burt finds 15% of his monthly maintenance fee by multiplying.

$$0.15 \times 912 = 136.80$$

The monthly property tax is \$136.80.

Multiply that by 12 to get the annual property tax.

$$12 \times 136.80 = 1,641.60$$

The annual property tax is \$1,641.60.

■ CHECK YOUR UNDERSTANDING

Maggie's monthly maintenance fee is m dollars, of which 27% is tax deductible for property tax purposes. Express the annual property tax deduction algebraically.

EXAMPLE 2

The Seaford Cove Cooperative is owned by the shareholders. The co-op has a total of 50,000 shares. Janet has an apartment at Seaford Cove and owns 550 shares of the cooperative. What percentage of Seaford Cove does Janet own?

SOLUTION Express Janet's shares as a fraction of the total number of shares. Janet owns $\frac{550}{50,000}$ of the corporation.

Divide.
$$\frac{550}{50,000} = 0.011$$

Janet owns 1.1% of the Seaford Cove Cooperative.

■ CHECK YOUR UNDERSTANDING

The Glen Oaks Village Co-op is represented by s shares. Sage owns r shares. Express the percent of shares he owns algebraically.

EXAMPLE 3

Gary Larson's job is relocating to a new city. He knows he will be there for at least 10 years. Gary is uncertain as to whether he should rent an apartment or buy a home for the time he will be working there. He knows that he eventually wants to return to his home city. Gary wants to compare the accumulated mortgage costs versus the accumulated rental costs before making a decision. Gary knows that he can afford a monthly rent of \$2,500. If he buys, he can put \$100,000 down and take out a \$350,000 mortgage for 20 years with an APR of 6%. Create a spreadsheet similar to the one created in Lesson 8-4 to assist Gary in making the comparison.

SOLUTION There are many factors that could enter into a comparison between renting and buying. In this case, you will only examine mortgage costs versus rental costs.

Gary's yearly rent increase can be modeled using an exponential function. Let R represent the yearly rent, A represent his initial annual rent, B represent the rate of increase expressed as a percent, and D represent the year number.

$$R = A \left(1 + \frac{B}{100} \right)^{D-1}$$

For example, suppose that Gary signs a contract for a monthly rent of \$800 and the annual rate of increase is 2%.

In the formula, A equals the amount paid annually for rent. When you substitute a value for A , multiply the monthly rent by 12. The total rent paid is

First year $R = 12 \cdot 800 \cdot (1 + 0.02)^{1-1} = 9,600 \cdot (1.02)^0 = \$9,600$

Second year $R = 12 \cdot 800 \cdot (1 + 0.02)^{2-1} = 9,600 \cdot (1.02)^1 = \$9,792$

Create a rental spreadsheet that has cells where the user can enter the initial monthly rent and the average yearly rent increase as shown in rows 2 and 3 in the spreadsheet on the next page.

■ CHECK YOUR UNDERSTANDING

Make a list. What other yearly costs might Gary have to consider for making this decision?

EXAMPLE 4

Jake and Gloria moved into an apartment and pay \$1,900 rent per month. The landlord told them that the rent has increased 4.1% per year on average. Express the rent y as an exponential function of the number of years they rent the apartment and determine the amount rent will be when they renew their lease for year 14.

SOLUTION Set up the exponential function you will enter where y is the monthly rent amount and x is the number of years Jake and Gloria have been renting.

$$y = 1,900(1 + 0.041)^{x-1}$$

Jake and Gloria's rent will not increase during the first year, so when $x = 1$, $(1 + 0.041)^{x-1}$ is equal to 1, so the rent is the initial rent of \$1,900.

Use the table feature on your graphing calculator to list the amount of rent each year. Because rent only increases one time a year (at the time of contract renewal), set the x -values of the tables to begin with 1 and only display whole number values for x .

To find the amount of rent charged in the 13th year, find the row in the table where $x = 13$. Jake and Gloria will pay \$3,077.20 each month of their 14th year in the apartment.

X	Y1	
9	2620.4	
10	2727.8	
11	2839.6	
12	2956	
13	3077.2	
14	3203.4	
15	3334.8	

X=13

■ CHECK YOUR UNDERSTANDING

In Example 4, suppose that the rent goes up \$60 per year. If y represents the rent and x represents the number of years, express Jake and Gloria's rent as a function of x .



EXAMPLE 5

The monthly rents for two-bedroom apartments at the luxury Cambridge Hall Apartments, for a 9-year period, are given in the table. Find and use an exponential regression equation to predict the rent in 2015.

SOLUTION Enter the data into your calculator. To make the calculations easier, use 2, 3, 4, 5, 6, 7, 8, 9, and 10 for the respective years.

The exponential regression equation with coefficients rounded to the nearest thousandth is

$$y = 2,185.288(1.051)^{x-1}$$

Substitute $x = 15$ to represent the year 2015.

$$y = 2,185.288(1.051)^{14}$$

The rent in 2015, rounded to the nearest cent, is predicted to be \$4,384.77.

Year	Monthly Rent (\$)
2002	2,425
2003	2,500
2004	2,675
2005	2,800
2006	2,950
2007	3,100
2008	3,250
2009	3,400
2010	3,575

■ CHECK YOUR UNDERSTANDING

Examine the regression equation from Example 5. To the nearest tenth of a percent, what was the approximate annual rent increase at Cambridge Hall Apartments?

EXAMPLE 6

In the 2000s the price of cooperative apartments soared, until the economic recession of 2009. In 1995, Ruth and Gino bought a co-op for \$98,000. They borrowed \$75,000 from the bank to buy their co-op. Years passed and they wanted to sell their co-op, but the price dipped to \$61,000. Their equity was \$6,744. If they sold the co-op, they would have to pay off the mortgage. How much money did they need to pay the bank back?

SOLUTION Ruth and Gino borrowed \$75,000 and paid the bank \$6,744 in principal. Subtract to find what they owed the bank.

$$75,000 - 6,744 = 68,256$$

They owed the bank \$68,256.

They could sell the co-op for \$61,000, but would have to pay the bank \$68,256. Subtract to find how much they would need to sell their co-op.

$$68,256 - 61,000 = 7,256$$

Ruth and Gino would have to add \$7,256 to pay the bank, since the price decreased.

■ CHECK YOUR UNDERSTANDING

Paul borrowed b dollars from a bank years ago when he bought his co-op for c dollars. He has built up equity and paid back d dollars towards his principal. The price dropped \$23,000 since he bought it. Write an inequality that expresses the fact that the new, decreased price of the co-op is less than what Paul owes the bank.

Applications

Home is a place you grow up wanting to leave, and grow old wanting to get back to.

John Ed Pearce, Journalist

1. What does this quote mean to you?
2. Three years ago, Jerry purchased a condo. This year his monthly maintenance fee is \$1,397. Twenty percent of this fee is for Jerry's property taxes. How much will Jerry pay this year in property taxes?
3. Last year, Anna paid x dollars for a co-op maintenance fee and one-third paid property taxes. How much property tax did Anna pay last year?
4. Ron has a co-op in Astor Cooperative. The total shares in the cooperative are 40,000 shares. If Ron owns 500 shares, what percentage of the cooperative corporation does he own?
5. The Jacobs Family owned a condo in Bethpage Acres. They bought it for \$130,000 six years ago and sold it last week for \$195,000. Who keeps the profit from the sale?
6. Ethel rented an apartment from a landlord in Sullivan County. Her rent was \$1,200 per month until she moved out last week. The new tenants pay \$1,350 per month. Represent the rent increase as a percent, to the nearest tenth of a percent.
7. Linda wants to purchase a Leisure Heights condominium apartment. She will borrow \$100,000 from the Dutchess Savings Bank. The bank is presently offering a 30-year fixed rate mortgage with an APR of 7.1%. Her monthly maintenance fee will be \$310.
 - a. What is the monthly mortgage payment to the nearest cent?
 - b. What will be her combined monthly payment?
8. The Basil family has a summer co-op apartment on a beach and pays a monthly mortgage payment of \$1,120 and a monthly maintenance fee of \$800. The Basils get approval from the co-op board of directors to *sublet* their apartment, since they do not plan to use it this summer. This means they will continue to make all the payments, but they will rent the apartment to a tenant for the three months they are away. The Basils will charge \$2,300 per month to the tenants.
 - a. What is the total of the three months' mortgage payments and maintenance fees?
 - b. How much rent will the Basils receive?
 - c. Will the rent cover the monthly fees that the Basils must pay?
 - d. What is the difference between the rent the Basils will collect and their monthly payments for the three months?
9. Helene and Vick moved into an apartment and pay \$1,875 rent per month. The landlord told them that the rent has increased 3.28% per year on average. Express the rent y as an exponential function of the number of years they rent the apartment.

- 10.** Andrew and Meghan moved into an apartment in the city and pay \$2,700 rent per month. The landlord told them the rent has increased 11.1% per year on average.
- Express the rent y as an exponential function of x , the number of years they rent the apartment. Show.
 - Suppose the rent has increased \$200 each year. Express the rent y as a function of the number of years x .
 - Graph the functions from parts a and b on the same axes.
 - Describe the difference between how the two graphs show the increase in rent.
- 11.** Maria borrowed \$120,000 from a bank when she bought her co-op for \$156,000. The price dropped x dollars since she bought it. She now owes the bank \$114,000, which is more than she could sell the co-op for. Write an inequality that expresses the fact that the new, decreased price of the co-op is less than what Maria owes the bank.
- 12.** Monthly rent at Countryside Co-ops has increased annually, modeled by the exponential equation $y = 12(2,155)(1.062)^{x-1}$. What was the percent increase per year?
- 13.** The monthly rent for a one-bedroom apartment at North Shore Towers for six consecutive years is shown in the table.
- Represent the years using the numbers 5, 6, 7, 8, 9, and 10 respectively. Draw a scatterplot for the data.
 - Find the exponential regression equation that models the rent increases. Round to the nearest thousandth.
 - Predict the rent in the year 2021. Round to the nearest dollar.
- 14.** The Tensers bought a mobile home for \$89,500. They rent space in a trailer park for \$900 per month. The rent increases 2% per year.
- If they put a down payment of \$10,000 on the trailer, how much must they borrow?
 - If they borrow the amount from part a for 15 years at an APR of 6%, what will the monthly payment be to the nearest cent?
 - What will be the first monthly payment? final monthly payment?
 - How much will they pay each month for their trailer and the space for the first year?
 - What will the space rental be to the nearest cent when they are making their final payment on the trailer?
- 15.** Joe wants to rent an apartment with an initial monthly rent of \$1,400. He has been told that the landlord raises the rent 1.25% each year. Set up an exponential function that models this situation. Calculate the rent after 12 years. Round to the nearest dollar.
- 16.** Use spreadsheets to compare these situations after 5 years.
- | | |
|---|--|
| total paid and total paid to the principal for a \$250,000, 20-year mortgage with a 5.75% APR | total amount paid for a \$2,100 monthly rent that has an annual increase of 1.5% |
|---|--|
- 17.** Use spreadsheets to compare these situations after 10 years.
- | | |
|--|---|
| total paid and total paid to the principal for a \$300,000, 15-year mortgage with a 6.5% APR | total amount paid for a \$2,600 monthly rent that has an annual increase of 2% after 10 years |
|--|---|

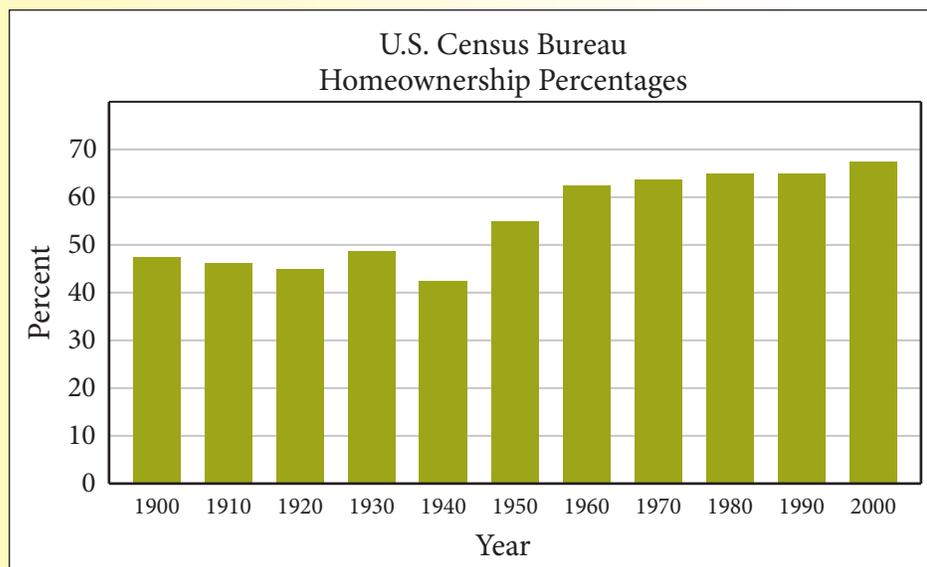
Year	Monthly Rent
2005	\$2,500
2006	2,590
2007	2,675
2008	2,750
2009	2,850
2010	2,925

Assessment

Real Numbers

You Write the Story!!

Examine the bar graph below. Write a short newspaper-type article centered on this graph. You can find an electronic copy of this graph at www.cengage.com/school/math/financialalgebra. Copy it, and paste it into your article. The graph depicts the historical census of housing from 1900 to 2000 in the United States.



Reality Check

1. Make a poster with columns labeled Less than \$100,000; \$100,000–\$200,000; \$200,000–\$300,000; and so on. Then, look in the real estate section of a local newspaper or a local online source. Cut out a picture or description of a house for each column of your poster. Give as much information about the dwelling as possible.
2. Search real estate websites for cities outside of your state. Print an image of a house that is priced in each of the categories on the poster from Exercise 1 above. Give as much information as possible. Compare the houses with those in your area.
3. Search for an online calculator that will determine payments required for mortgages. Enter a value for a house, an interest rate, and a payment term, and calculate the monthly payment. Figure the total amount you would pay, including interest, for the house.
4. Search for an online calculator that will determine mortgage payments. Experiment with different loan amounts and different mortgage terms. Can you determine a pattern for the increase in your monthly payment for each additional \$1,000 that you borrow?

5. Design the first floor of a house. Make a scale drawing using a scale of $\frac{1}{4}$ inch = 1 foot. Include bedrooms, bathrooms, a kitchen, a living room, and anything else you choose.
6. Visit a real estate broker. Find information on homes for sale in your area including price range of homes, average price of a home, average price of a rental, average price of a co-op or condo, and range of property taxes. Prepare a report that includes your findings.
7. Visit your local bank. Find out information about mortgage loans they offer. Determine the rates for a fixed-rate mortgage and an adjustable-rate mortgage. Ask about the required down payment.
8. Interview a lawyer who represents clients at closings. Ask the lawyer to describe a typical closing and what prospective buyers need so that the closing goes smoothly. Prepare a report of your findings.
9. Search for an online calculator that will generate an amortization table when you make extra payments each month. Use the calculator to determine the difference in total interest on a \$500,000, 20-year mortgage with an APR of 7% when the homeowner makes monthly extra payments of \$50, \$100, and \$200.
10. Search online for examples and prices of modular housing. Make a list of advantages and disadvantages of modular housing as compared to houses built on-site. Prepare a report that includes any other facts and figures you find out about modular housing.
11. Search online for prices, fees, rules, and regulations related to trailer parks and mobile homes. List advantages and disadvantages of each. Find facts and figures about trailer park locations, prices, and popularity. Restrict your investigation to year-round trailer-park housing, not RV campgrounds and vacation parks. Prepare a report.
12. Talk to your teacher about planning a class debate on the issue of renting versus buying a home. Split the class into two teams and have each team do research. Have your teacher moderate the debate and formulate questions to be addressed by each side.
13. Determine a comparable monthly rent and mortgage payment. Find an apartment and a home you could rent and buy for that amount. Determine the current rate for a 15-year mortgage and the yearly rent increase percentage for your area (look online or ask a real estate broker). Run the numbers for a 5-, 10-, and 15-year stay in each. Make a display board about your findings.
14. Go online and find how property tax is determined where you live. Search for property tax in your city or county. Prepare a poster that describes the assessment and property tax computation procedures.



Dollars and Sense

Your Financial News Update

Go to www.cengage.com/school/math/financialalgebra where you will find a link to a website containing current issues about home purchases, rentals, condos, and co-ops. Try one of the activities.

At the beginning of the chapter, you were given the weights of the heaviest buildings ever moved. The Fu Gang Building in China was quoted in metric tones. A metric ton is about 10% heavier than an English Standard ton. Let's try to get a feeling for how much that building really did weigh.

1. How many pounds are in an English Standard ton?
2. What do you get when you increase that amount by 10%?
3. The answer to Exercise 2 is the approximate number of pounds in a metric ton. If the Fu Gang Building weighed approximately 15,140 metric tons, what is its equivalent weight in pounds?
4. Assuming that the average weight of an elephant is 9,000 pounds, what was the equivalent weight of the Fu Gang Building in average elephants?

Applications

1. A rectangular room measures 18 feet by 25 feet. It is going to be carpeted with carpeting that sells for \$8.45 per square foot.
 - a. What is the area of the room in square feet?
 - b. If the room is drawn to a scale of $\frac{1}{4}$ inch represents 1 foot, give the dimensions of the room in inches on the scale drawing.
 - c. This room is well-insulated and on the south side of the house. It has an 8-foot-high ceiling. How large an air conditioner would this room require? Round to the nearest thousand BTUs.
2. Ricky took out a \$268,000, 30-year mortgage at an APR of 6.34%.
 - a. What is the monthly payment to the nearest cent?
 - b. What will be his total interest charges after 30 years, to the nearest thousand dollars?
3. Adam is taking out a \$197,000 mortgage. His bank offers him an APR of 7.45%. He wants to compare monthly payments on a 20- and a 30-year loan. Find, to the nearest ten dollars, the difference in the monthly payments for these two loans.
4. Eduardo owns a condominium. This year his monthly maintenance fee is m dollars. Twenty-seven percent of this fee pays for Eduardo's property taxes, and 11% pays for the mortgage on the entire development. Both of these expenses are tax-deductible. Express the amount that is tax-deductible algebraically.
5. A gazebo in the shape of a regular decagon (10 sides) has side length s and apothem a . Express the area of the floor A , algebraically.

6. Brianna just signed a lease on a rental apartment. The current rent is \$1,330 per month, and she estimates a 6% increase each year. Use her estimate to predict the sum of the next five years' worth of monthly rental expenses. Round to the nearest thousand dollars.
7. The Bricely Family borrowed \$176,000 from Glen Bank several years ago when they bought their co-op for \$246,000. The price dropped d dollars since they bought it. After making years of payments and paying some of the principal, they now owe the bank b dollars, which is more than the price for which they could sell the co-op.
 - a. Write an inequality that expresses the fact that the new, decreased price of the co-op is less than what the Bricelys owe the bank.
 - b. Express the amount of extra money the Bricelys need to raise to pay the bank if they wanted to sell their co-op for d dollars less than the price for which they could sell the co-op.
8. The Maxwell family took out a \$275,000, 20-year mortgage at an APR of 6.1%. The assessed value of their house is \$9,400. The annual property tax rate is 90.82% of assessed value. What is the annual property tax?
9. The market value of a home is \$311,000. The assessed value is x dollars. The annual property tax rate is a dollars per \$1,000 of assessed value. Express the semi-annual property tax bill algebraically.
10. Katherine and Alex had an adjusted gross income of g dollars. Katherine just got a \$2,000 raise. They are considering moving to a new house with monthly mortgage payment m dollars, semiannual property taxes s dollars, and quarterly homeowner's premium q dollars. Express their front-end ratio algebraically.
11. The Xiomaras have an adjusted gross income of \$137,865. They are looking at a new house that would have a monthly mortgage payment of \$1,687. Their annual property taxes would be \$7,550 and their semi-annual homeowner's premium would be \$835.
 - a. Find their front-end ratio to the nearest percent.
 - b. Assume that their credit rating is good. Based on the front-end ratio, would the bank offer them a loan? Explain.
 - c. The Xiomaras have a \$344 per month car loan, and their average monthly credit card bill is \$420. Compute the back-end ratio to the nearest percent.
 - d. Based on the back-end ratio, would the bank offer them a loan? Explain.
12. Lexi moved into an apartment in the suburbs and pays \$1,975 rent per month. The landlord told her that the rent has increased 3.6% per year on average. Express the rent y as an exponential function of x , the number of years she rents the apartment.
13. Harley built a concrete patio in her backyard. It is a free-form shape and she needs to find the area of it for property tax purposes.
 - a. She takes a diagram of the patio and places it inside a 30-ft by 25-ft rectangle. What is the area of the rectangle?
 - b. She then has a graphing calculator generate 20,000 random points inside the rectangle. She finds that 12,451 of these points landed in the patio outline. What percent of the points landed in the patio? Round to the nearest percent.
 - c. What is the area of the patio, to the nearest square foot?

- 14.** Ivana Chase has a gross bimonthly income of \$2,900. She pays 16% in federal and state taxes, puts aside 12% of her income to pay off her school loan, and puts 5% of her income aside for savings. She is considering an apartment that will rent for \$1,700 per month.
- Does this monthly rental fee fall within the recommended 25–30% housing expense range?
 - Based upon her expenses, can she make the monthly payments? How much will remain after she pays for the rent and other expenses?
- 15.** The square footage and monthly rental of 15 similar two-bedroom apartments in Martha's Cove yield the following linear regression formula: $y = 1.137x + 598.98$ where x represents the square footage of the apartment and y represents the monthly rental price.
- Use the formula to determine the monthly rent for an apartment that has 1800 square feet.
 - Using the recommendation that you should spend no more than 28% of your monthly gross income on housing, can Stephanie afford this rental if she makes \$9,800 per month? Explain.
- 16.** Johnny took out a \$500,000 30-year mortgage with an APR of 6.95%. The first month he made an extra payment of \$1,000. What was his balance at the end of that first month?
- 17.** James rents an apartment with an initial monthly rent of \$1,600. He was told that the rent goes up 1.75% each year. Write an exponential function that models this situation to calculate the rent after 15 years. Round the monthly rent to the nearest dollar.
- 18.** Elizabeth is moving from a one bedroom apartment in one city to a similar apartment in another city. She has been quoted a flat fee for the truck rental and has two estimates for wages of the movers she will hire depending on her needs and when she moves.

Weekday Move	Weekend Move
5 hours of loading and unloading services	4 hours of loading and unloading services
4 hours of packing and unpacking services	5 hours of packing and unpacking services
\$730 total cost	\$710 total cost

Luke's Moving Company charges a set hourly moving team rate for loading and unloading, and a different set hourly moving team rate for packing and unpacking. Determine the hourly rates.

- 19.** Fill in the missing entries in this loan amortization table for a \$220,000 20-year mortgage with an APR of 5.95%.

Payment Number	Beginning Balance	Monthly Payment	Towards Interest	Towards Principal	Ending Balance
1	220,000.00	a.	1,090.83	478.98	b.
2	b.	a.	1,088.46	481.35	219,039.67
3	219,039.67	a.	c.	483.74	218,555.94
4	218,555.94	a.	1,083.67	d.	218,069.80
5	218,069.80	a.	1,081.26	488.55	e.

20. Joanne and Matt have been approved for a \$350,000, 15-year mortgage with an APR of 6.25%. Using the mortgage and interest formulas, set up a two-month amortization table with the following headings and complete the table for the first two months.

Payment Number	Beginning Balance	Monthly Payments	Towards Interest	Towards Principal	Ending Balance
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21. Use the spreadsheets from Lessons 8-4 and 8-5 to compare the following two situations after an 8-year period.

total amount paid and total amount paid to principal for a \$250,000, 18-year mortgage with an APR of 6.35%	total amount paid for a \$1,700 monthly rent that has an annual increase of 1.8%
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22. Use the spreadsheets from Lessons 8-4 and 8-5 to compare the following two situations after a 7-year period.

total amount paid and total amount paid to principal for a \$370,000, 20-year mortgage with an APR of 7.1%	total amount paid for a \$2,600 monthly rent that has an annual increase of 2%
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23. Michelle took out a \$370,000 30-year adjustable rate mortgage with a 3.8% initial 6-month rate. The amortization table for the initial rate period is shown. After the first 6 months, the rate went up to 4.8%. Calculate the next line of the amortization table.

Payment Number	Beginning Balance	Monthly Payment	Towards Interest	Towards Principal	Ending Balance
1	370,000.00	1,724.04	1,171.67	552.38	369,447.62
2	369,447.62	1,724.04	1,169.92	554.12	368,893.50
3	368,893.50	1,724.04	1,168.16	555.88	368,337.62
4	368,337.62	1,724.04	1,166.40	557.64	367,779.98
5	367,779.98	1,724.04	1,164.64	559.41	367,220.57
6	367,220.57	1,724.04	1,162.87	561.18	366,659.40

24. Calculate the missing amounts in the amortization table which shows extra payments toward the principal made each month.

Payment Number	Beginning Balance	Monthly Payment	Towards Interest	Towards Principal	Ending Balance	Extra Payment
1	210,000.00	1,628.13	1,225.00	403.13	a.	100
2	a.	1,628.13	1,222.07	b.	208,890.81	200
3	208,890.81	1,628.13	c.	409.60	208,381.21	100
4	208,381.21	1,628.13	1,215.56	412.57	207,918.64	d.
5	207,918.64	1,628.13	1,212.86	415.27	e.	100